A Near-Optimum Matching Section Without Discontinuities

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Abstract—**The optimum tapered matching section requires impedance steps at the taper ends, which exclude in most cases its application for synthesizing waveguide tapers. A new "near"-optimum design is described which avoids the impedance steps and yields tapers only a fractional part longer in taper length (or narrower in bandwidth) than the exactly optimum taper. A table of values of a transcendental function has been calculated to simplify the synthesis. An example compares both types of matching sections.**

I. Introduction

The APPLICATION of the optimum design of transmission-line tapers as given by Klopfenstein [1] to the synthesis of waveguide matching sections is often limited. The reason for this lies in impedance discontinuities at the taper ends which are inherent in the optimum taper (sometimes referred to as the Dolph–Chebyshev Taper). Therefore, the use of such a design is inadvisable if excitation of spurious modes must be avoided. In addition, the impedance steps in general also introduce reactances which may have to be compensated, thus complicating the design.

It is the intention of this paper to demonstrate a "near"-optimum taper without steps in the impedance. As usual, the optimum taper has maximum bandwidth for a given length (or minimum length for a given bandwidth) *and* for a specified magnitude of the reflection coefficient in the passband.

A new approach to the solution of the nonlinear differential equation for the reflection coefficient yields more accurate results because the solution is not restricted to the passband or to small ratios of the impedances to be matched as long as the taper remains gradual. The tapered sections of this design are only a fractional part longer in taper length (or narrower in bandwidth) compared with the exactly optimum taper. Their reflection coefficient can easily be computed using ordinary trigonometric and hyperbolic functions. The synthesis may be simplified with the use of a table of values of a transcendental function. An example will be given for the case of a simple transmission-line matching section and will be compared to the optimum taper.

II. Reflections in Nonuniform Waveguides and Transmission Lines

In Fig. 1, a section of nonuniform waveguide is shown together with the coordinates used in the following theory. Solymar [2] has shown that the electromagnetic field inside the waveguide at any cross section can be represented by forward and backward traveling waves A and B, respectively, of all normal modes of that cross

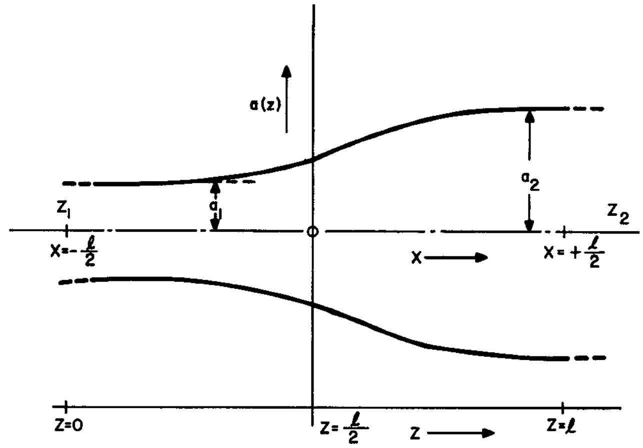

Fig. 1. A nonuniform waveguide section.

section, provided none of the wave impedances become singular or zero. These waves are then interrelated by an infinite set of coupled differential equations. We assume that the nonuniform waveguide is primary excited by only one mode, the main mode, and that it has a much larger amplitude than all other modes. In this case, one may neglect all unwanted modes and with the complex reflection coefficient

$$R = \frac{B}{A}$$

one can derive a nonlinear differential equation in R:

$$\frac{dR}{dz} - 2j\beta_1(z)R + (1 - R^2)F_1(z) = 0 \tag{1}$$

in which $\beta_1(z)$ is the propagation constant of the mode under consideration. The function $F_1(z)$ represents a local reflection coefficient per unit length and is related to the wave impedance $Z_1(z)$ of the main mode as follows:

$$F_1(z) = \frac{1}{2} \frac{d \log_e Z_1(z)}{dz}. \tag{2}$$

The same equation (1) can be found for the TEM mode in lossless two-conductor transmission-line matching sections if again higher order modes are neglected [4]. In this special case

$$Z_1(z) = Z_c = \sqrt{\frac{L(z)}{C(z)}} \tag{3a}$$

$$\beta_1(z) = \omega\sqrt{L(z)C(z)} \tag{3b}$$

and hence

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$$F_1(z) = \frac{1}{2}\frac{d}{dz}\left\{\log_e \sqrt{\frac{L(z)}{C(z)}}\right\} \tag{4}$$

where $L(z)$ is the distributed series inductance of the line per unit length and $C(z)$ is the distributed shunt capacity per unit length.

Equation (1) is a first-order Riccati equation for which a general closed-form solution is not known. Approximate solutions can be found if $R^2 \ll 1$. The optimum design of transmission-line tapers is based on this assumption [1], [3]. In the following, we shall consider a different approximate solution for which we make first the coordinate transformation

$$x = z - \frac{l}{2} \tag{5}$$

in order to obtain results easy to compare with those given by Klopfenstein [1]. Next we assume the reflection coefficient $\rho(x) = R(z)$ to be of the form

$$\rho(x) = r(x)e^{j2\psi(x)} \tag{6}$$

where $r(x)$ and $\psi(x)$ are real functions of x. Using this transformation in (1) results in two coupled nonlinear differential equations:

$$\frac{dr}{dx} = -F(x)\cos 2\psi(x)(1 - r^2) \tag{7}$$

and

$$2r\left(\frac{d\psi}{dx} - \beta(x)\right) = F(x)\sin 2\psi(x)(1 + r^2) \tag{8}$$

where $\beta(x) = \beta_1(z)$ and $F(x) = F_1(z)$.

If $|r| \neq 1$, we find from (7) after separation of the variables and integration

$$\operatorname{arctanh}(r) = -\int_{-l/2}^{x} F(x')\cos 2\psi(x')dx' + \operatorname{arctanh}\left[r\left(-\frac{l}{2}\right)\right]. \tag{9}$$

Let us replace the integral in (9) by a new variable $y(x)$:

$$y(x) = -\int_{-l/2}^{x} F(x')\cos 2\psi(x')dx'. \tag{10}$$

Then $r(x)$ is given by

$$r(x) = \tanh\left\{y(x) + \operatorname{arctanh}\left[r\left(-\frac{l}{2}\right)\right]\right\}. \tag{11}$$

It follows further from (8)

$$\psi(x) - \psi\left(-\frac{l}{2}\right) = \int_{-l/2}^{x}\beta(x')dx' + \frac{1}{2}\int_{-l/2}^{x} F(x')\sin 2\psi(x')\left(\frac{1 + r(x')^2}{r(x')}\right)dx' \tag{12}$$

for $r \neq 0$.

The waveguide taper exhibits the particular problem that, at frequencies close to cutoff, the propagation constant β and the wave impedance Z vary strongly along the tapered section. Thus near or at cutoff, there is no further simplification or approximation possible, and (10) and (12) have to be solved numerically precluding a direct synthesis. Above cutoff, however, a first approximation would be to neglect the perturbational term in (12) if the taper is gradual. In this case (12) becomes

$$\psi(x) = \psi\left(-\frac{l}{2}\right) + \int_{-l/2}^{x}\beta(x')dx' \tag{12a}$$

and with

$$\psi\left(-\frac{l}{2}\right) = \psi_0 = -\beta_0\frac{l}{2}, \qquad \left(\beta_0 = \beta\left(-\frac{l}{2}\right)\right)$$

(10) can be written as

$$y = -\operatorname{Re}\left\{\int_{\psi_0}^{\psi}\left(\frac{F}{\beta}\right)e^{-2j\psi'}d\psi'\right\}$$

with ψ now to be considered an auxiliary variable. Even this approximation is not particularly suited for the synthesis of a waveguide taper [5], and one may have to assume further that β is independent of x. By doing this the design of a waveguide taper is similar to the case of the transmission-line taper, which will be considered in the next paragraphs.

The Tapered Transmission-Line Matching Section

The assumption of a TEM mode in a homogeneous medium implies that β is independent of x but dependent on ω; thus β will go to zero for all x if ω goes to zero, i.e., if we are at very low frequencies. A solution of (12) for ψ is then with $\psi_0 = 0$

$$\psi(x) = 0, \qquad \text{for } \omega = 0.$$

Hence we obtain from (10) the function $y(x)$:

$$y(x) = -\int_{-l/2}^{x} F dx', \qquad \text{for } \omega = 0.$$

From (11) with $r[+(l/2)] = 0$ and observing that the impedance $Z_c[-(l/2)] = Z_1$ and $Z_c[+(l/2)] = Z_2$, the input reflection coefficient at $x = -(l/2)$ is thus found to be

$$r\left(-\frac{l}{2}\right)\Bigg|_{\omega=0} = r_0 = \tanh\left\{\log_e\sqrt{\frac{Z_2}{Z_1}}\right\} \tag{13}$$

which is equivalent to

$$r_0 = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{14}$$

as must be. Thus the transformation according to (11) yields for $\omega = 0$ the correct result. On the other hand, (12a) becomes now for $\omega > 0$

$$\psi(x) \cong \psi_0 + \beta x + \beta\frac{l}{2}.$$

Introducing $\psi(x)$ into (10) and since $\psi_0 = -\beta(l/2)$, we obtain finally as a first approximation for $y(x)$:

$$y(x) = - \text{Re} \left\{ \int_{-l/2}^{x} F(x') e^{-j2\beta x'} dx' \right\}.$$

With $r[+(l/2)]=0$, the input reflection coefficient becomes, therefore:

$$r\left(-\frac{l}{2}\right) = \tanh \text{Re} \left\{ \int_{-l/2}^{+l/2} F(x) e^{-j2\beta x} dx \right\}. \quad (15)$$

It is worthwhile to note that, if ω approaches zero, this approximation still yields the correct value for the reflection coefficient as given by (14).

III. The Design of a Tapered Matching Section

In many practical cases, the design of a tapered matching section is based upon the requirement that the magnitude of the reflection coefficient at the input be below a specified value. Since the magnitude of the reflection coefficient r is a monotonic function of y, one is interested to find a function $F(x)$ which keeps for $r[+(l/2)]=0$ the function

$$y\left(\frac{l}{2}\right) = - \text{arctanh}\, r\left(-\frac{l}{2}\right) = - \text{Re} \left\{ \int_{-l/2}^{+l/2} F(x) e^{-j2\beta x} dx \right\} \quad (16)$$

below a required value determined by the specification for $r[-(l/2)]$.

By analogy with the theory outlined in [1] and [3], the integral

$$f(\beta) = \int_{-l/2}^{+l/2} F(x) e^{-j2\beta x} dx \quad (17)$$

will be considered as the Fourier transform of $F(x)$. Hence $F(x)$ is given by

$$F(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(\beta) e^{+j2\beta x} d2\beta. \quad (18)$$

Once $f(\beta)$ is specified, $F(x)$ can be calculated from (18) and $\log_e [Z(x)]$ is obtained by integrating $F(x)$.

The Optimum Taper

The optimum design of a tapered transition (i.e., a taper having maximum bandwidth for a given magnitude of the reflection coefficient) is obtained if $f(\beta)$ follows the relation [1]:

$$f(\beta) = y_0 \cdot \frac{\cos \sqrt{(\beta l)^2 - A^2}}{\cosh A} \quad (19)$$

sometimes referred to as the Dolph–Chebyshev response. Defining the ripple function

$$\epsilon = \frac{\cos \sqrt{(\beta l)^2 - A^2}}{\cosh A} \quad (20)$$

which becomes unity for $\beta l = 0$ and has equal maxima and minima $\pm (\cosh A)^{-1}$ for $\beta l \geq A$, then A specifies the amount of ripple in the passband beginning at $\beta l = A$. Since $f(\beta)$ is real and $y(l/2) = -f(\beta)$, the amplitude of the reflection coefficient becomes with (15), (17), (19), and (20):

$$r\left(-\frac{l}{2}\right) = \tanh (y_0 \epsilon). \quad (21)$$

The value of y_0 has to be determined such that (17) is satisfied for all values of β, including $\beta = 0$. From (19) it follows

$$y_0 = \lim_{\beta \to 0} \{f(\beta)\}$$

and using (17) one finds

$$y_0 = \log_e \sqrt{\frac{Z_2}{Z_1}} \quad (22)$$

which again yields the natural value for r_0, as given by (14). Thus this specification (19) of $f(\beta)$ and hence according to (16) the specification of y will describe also for low frequencies and large ratios Z_2/Z_1 the "correct" reflection coefficient. In fact, the ratio Z_2/Z_1 may have any value provided the taper is sufficiently long such that $\log_e Z(x)$ and $\beta(x)$ change very little with x.

The physical contour of the transition requires the knowledge of the Fourier transform $F(x)$ of (19), which can be found in [3]. Integrating $F(x)$ yields $\log_e Z(x)$, which, due to impulse functions in $F(x)$, will contain step discontinuities at $x = \pm (l/2)$. These steps in the impedance are particularly inconvenient in tapered sections of waveguide because they introduce reactances which degrade the performance of the taper and may have to be compensated. Apart from this they will excite undesired spurious modes.

A Near-Optimum Taper without Impedance Steps

There exists, however, a specification which yields an almost optimum tapered section without discontinuities in the impedance. The penalty for the elimination of the steps at the taper ends lies in a small but definite increase in taper length for equal bandwidth if compared with the optimum taper. This design is derived from a modification of the ideal Dolph–Chebyshev response (19) yielding a new specification for $f(\beta)$:

$$f(\beta) = y_0 \frac{\cos \sqrt{(\beta l)^2 - A^2} - \cos \sqrt{(\beta l)^2 - B^2}}{\cosh A - \cosh B}$$

which is a superposition of two different functions as given by (19). Since we know that each individual term of the numerator has a Fourier transform, we must expect that the impulse functions cancel out due to the negative sign of the second term in the numerator.

It is easily seen that $f(\beta)$ has again the value y_0 at $\beta l = 0$ and oscillates for increasing βl around $f(\beta) = 0$ with continuously varying extreme values. The largest extreme value in the passband must be considered as the most essential parameter of $f(\beta)$, which should be as close as possible to the origin $\beta = 0$ in order to obtain

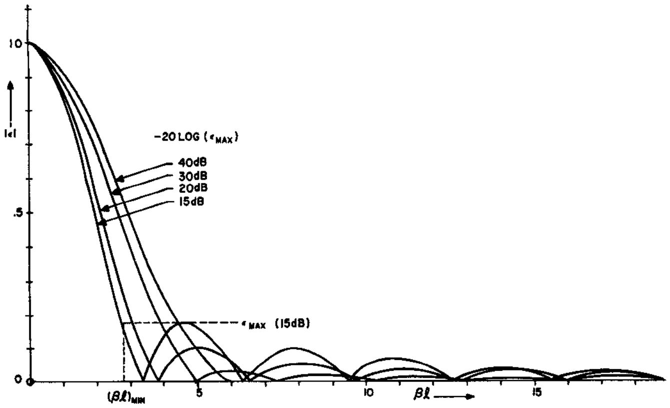

Fig. 2. Magnitude of ripple function ϵ versus βl of near-optimum taper.

minimum taper length for a given lowest frequency limit. The location of the extreme values is found by differentiating $f(\beta)$ with respect to (βl) and checking the resulting differential for zeros. At $\beta l=0$ we find the first maximum y_0 (if $y_0 \neq 0$), which will be considered as the main lobe. For subsequent lobes it can be shown that, for any fixed value of B where $B<A$, all extreme values shift towards the origin if A approaches B. In the limiting case, if $A=B$, we obtain the modified Dolph–Chebyshev response[1]

$$f(\beta) = y_0 \frac{B}{\sinh B} \frac{\sin \sqrt{(\beta l)^2 - B^2}}{\sqrt{(\beta l)^2 - B^2}} \cdot \tag{23}$$

Inspection of this new specification for $f(\beta)$ indicates that the first sidelobe of the ripple function

$$\epsilon = \frac{B}{\sinh B} \frac{\sin \sqrt{(\beta l)^2 - B^2}}{\sqrt{(\beta l)^2 - B^2}} \tag{24}$$

is the largest in magnitude and is solely dependent on the parameter B. Therefore, B is the new ripple parameter. Once this value and the lower frequency limit are fixed, the minimum length of the matching section is known. For illustration the magnitude of ϵ is plotted in Fig. 2 as function of βl for some typical B values. For arbitrary B, the first maximum of (24) ($\beta l>0$) has the value

$$\epsilon_{\max} = \frac{B}{\sinh B} \quad (0.21723) \tag{25}$$

which determines the maximum ripple of the reflection coefficient in the passband:

$$r_{\max} = \tanh (y_0 \epsilon_{\max}). \tag{26}$$

The minimum taper length is found by setting (24) and (25) equal and solving for minimum βl. This results in

$$(\beta l)_{\min} = \sqrt{B^2 + 6.523}. \tag{27}$$

In case the value B becomes zero, (23) represents the response of the exponential taper [7], and for maximum allowable reflection coefficients larger than tanh $(0.21723 y_0)$, this taper may, therefore, be used as a matching section. Fig. 3 shows B and $(\beta l)_{\min}$ as a function of $\epsilon_{\max}$, including the case of the exponential taper. Since the parameter A in (19) determines the minimum length of the optimum taper, A has been calculated as a function of the passband ripple and is also shown in Fig. 3 as a dashed curve. Comparing A and $(\beta l)_{\min}$, we find that the minimum taper length of the modified taper at most is about 14 percent longer than the optimum taper. This occurs where the modified taper becomes identical with the exponential taper. For smaller ripples, the relative increase is smaller.

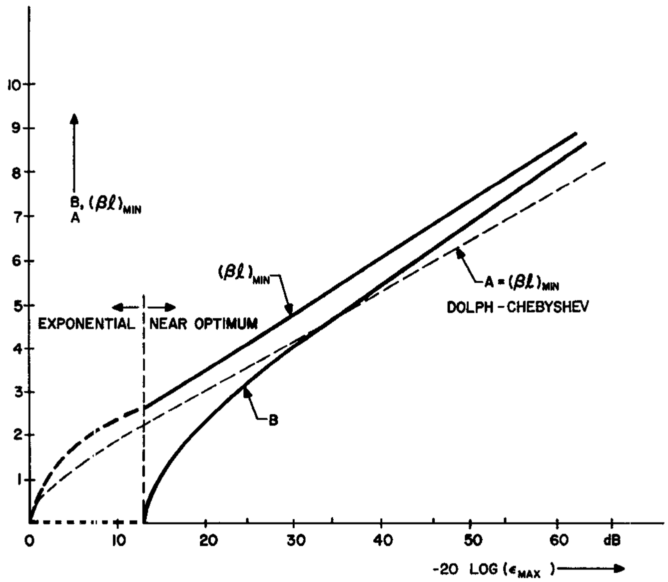

Fig. 3. B, $(\beta l)_{\min}$ and A versus maximum ripple in the passband.

The physical dimensions of the matching section are again found with the Fourier transform of the new function $f(\beta)$ (23), which becomes [6]

$$F(x) = y_0 \frac{B}{\sinh B} \frac{1}{l} I_0 \left\{ B \sqrt{1 - \left(\frac{2x}{l} \right)^2} \right\}, \quad \text{for } |x| < \frac{l}{2}$$

and

$$F(x) = 0, \quad \text{for } |x| > \frac{l}{2} \tag{28}$$

where $I_0(z)$ represents the modified Bessel function of the first kind. As has been expected, the specification by (23) avoids the impulse function in $F(x)$ at the ends of the tapered section.

The next step to obtain $Z_c(x)$ will be the integration of (28), and we find after some algebra

$$\log_e Z_c(\xi) = \tfrac{1}{2} \log_e (Z_2 Z_1) + \tfrac{1}{2} \log_e \left(\frac{Z_2}{Z_1} \right) G(B, \xi) \tag{29}$$

where $\xi = 2x/l$. The function $G(B, \xi)$ is given by

[1] It turns out that this function is known in antenna theory as "modified pin x/x response" and was first introduced by Taylor [8].

TABLE I

VALUES OF THE FUNCTION $G(B, \xi)$; $\xi = 2x/l$

$20 \log_{10} (1/\epsilon_{max})$	15	20	25	30	35	40	45	50	55	60
B	1.1177	2.3204	3.2136	4.0091	4.7552	5.4710	6.1663	6.8466	7.5154	8.1752
$(\beta l)_{min}$	2.7875	3.4503	4.1046	4.7533	5.3975	6.0376	6.6741	7.3073	7.9374	8.5648
ξ					$G(B, \xi)$					
0	0	0	0	0	0	0	0	0	0	0
0.50	0.05473	0.06607	0.07514	0.08279	0.08950	0.09553	0.10106	0.10620	0.11102	0.11558
0.10	0.10939	0.13186	0.14979	0.16487	0.17806	0.18989	0.20071	0.21074	0.22012	0.22897
0.15	0.16390	0.19708	0.22345	0.24554	0.26476	0.28194	0.29757	0.31200	0.32545	0.33809
0.20	0.21819	0.26146	0.29565	0.32412	0.34874	0.37060	0.39039	0.40854	0.42535	0.44105
0.25	0.27218	0.32472	0.36593	0.39998	0.42919	0.45492	0.47802	0.49905	0.51838	0.53630
0.30	0.32580	0.38661	0.43387	0.47254	0.50539	0.53405	0.55953	0.58249	0.60341	0.62661
0.35	0.37899	0.44687	0.49907	0.54129	0.57673	0.60729	0.63415	0.65807	0.67961	0.69917
0.40	0.43166	0.50526	0.56117	0.60578	0.64272	0.67413	0.70135	0.72528	0.74654	0.76558
0.45	0.48375	0.56158	0.61986	0.66566	0.70298	0.73421	0.76085	0.78390	0.80406	0.82184
0.50	0.53519	0.61560	0.67487	0.72063	0.75726	0.78735	0.81255	0.83397	0.85237	0.86832
0.55	0.58591	0.66715	0.72597	0.77052	0.80544	0.83355	0.85660	0.87580	0.89196	0.90568
0.60	0.63586	0.71607	0.77302	0.81521	0.84754	0.87296	0.89333	0.90990	0.92352	0.93483
0.65	0.68495	0.76220	0.81587	0.85469	0.88369	0.90589	0.92321	0.93694	0.94794	0.95683
0.70	0.73314	0.80543	0.85449	0.88904	0.91412	0.93276	0.94688	0.95774	0.96618	0.97281
0.75	0.78036	0.84566	0.88886	0.91840	0.93916	0.95410	0.96504	0.97317	0.97928	0.98391
0.80	0.82655	0.88282	0.91901	0.94298	0.95924	0.97051	0.97846	0.98413	0.98824	0.99123
0.85	0.87166	0.91684	0.94505	0.96307	0.97483	0.98265	0.98940	0.99152	0.99401	0.99574
0.90	0.91564	0.94771	0.96710	0.97901	0.98646	0.99117	0.99419	0.99615	0.99743	0.99827
0.95	0.95844	0.97543	0.98534	0.99119	0.99466	0.99674	0.99800	0.99876	0.99923	0.99952
1	1	1	1	1	1	1	1	1	1	1

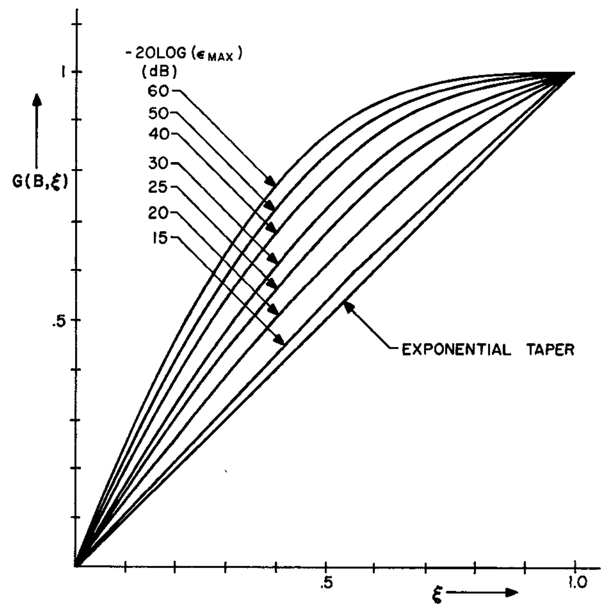

Fig. 4. Function $G(B, \xi)$ according to (30) versus ξ, $\xi = 2x/l$.

$$G(B, \xi) = \frac{B}{\sinh B} \int_0^{\xi} I_0\{B\sqrt{1-\xi'^2}\}\, d\xi'. \quad (30)$$

Note that $G(B, \xi) = -G(B, -\xi)$.

The transcendental function $G(B, \xi)$ is plotted in Fig. 4 and tabulated in Table I for some practical values of the parameter B.

IV. EXAMPLE

As an illustrative example, a section to match a 50- to 75-Ω transmission line will be designed to have a return loss not lower than 40 dB, that is, $r_{max} = 0.01$.

With (21), the reflection coefficient at dc ($\beta l = 0$) becomes $r_0 = 0.2$, which corresponds to a return loss of about 14 dB.

Since with (22) $y_0 = 0.2027$, we have to require an additional return loss of about 26 dB represented in (26) by

TABLE II

$\lvert\xi\rvert = \lvert 2x/l \rvert$	$G(B, \xi)$	$Z_c(+\lvert\xi\rvert)$ Ω	Z^a Ω	$Z_c(-\lvert\xi\rvert)$ Ω	Z^a Ω
0	0.0	61.237	61.237	61.237	61.237
0.1	0.15281	63.164	62.960	59.369	59.561
0.2	0.30134	65.095	64.690	57.608	57.969
0.3	0.44160	66.973	66.382	55.993	56.491
0.4	0.57009	68.740	67.996	54.553	55.150
0.5	0.68402	70.346	69.492	53.308	53.963
0.6	0.78146	71.750	70.936	52.265	52.939
0.7	0.86140	72.922	72.006	51.425	52.079
0.8	0.92380	73.850	72.986	50.778	51.380
0.9	0.96948	74.537	73.774	50.310	50.831
1.0	1	75.000	74.376	50.000	50.420

[a] Characteristic impedance of Dolph–Chebyshev Taper.

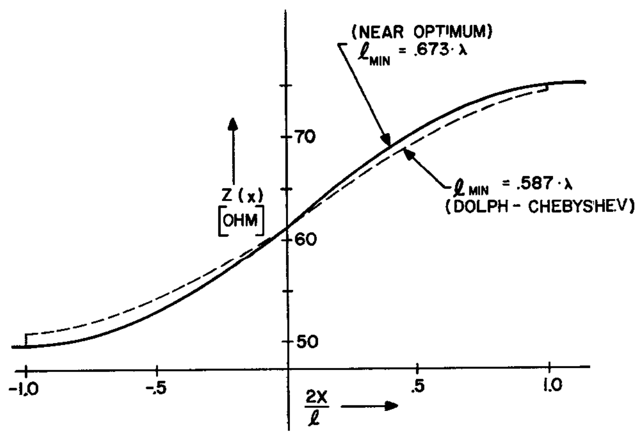

Fig. 5. Characteristic impedance versus length of 50–75-Ω matching sections.

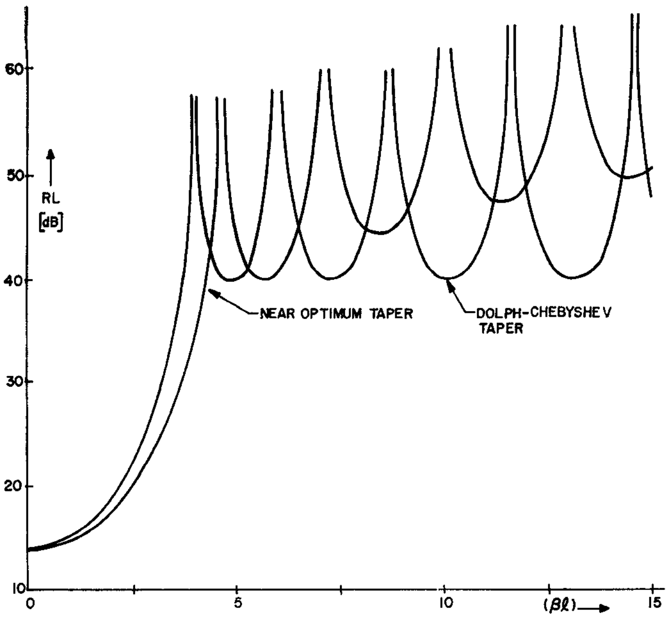

Fig. 6. Return loss versus (βl) of 50–75-Ω matching sections.

$\epsilon_{max} \cong 0.0493$. Using Table I, we first obtain by linear interpolation

$$B = 3.3727 \quad \text{and} \quad (\beta l)_{min} = 4.2302.$$

$(\beta l)_{min}$ determines the minimum length for given lowest frequency, whereas B determines with $G(B, \xi)$ the impedance variation. Table II shows values of the characteristic impedance Z_c along the x coordinate of the near-optimum taper as computed with the aid of Table I and by linear interpolation. The error made due to the interpolation is less than 0.2 percent.

Fig. 5 depicts the impedance variation along the axis of the transmission line for the optimum and near-optimum taper. It is worthwhile to note that the near-optimum taper needs a minimum length of 0.6733 λ, whereas the value for the optimum taper is 0.5871 λ. This is an increase by only less than 13 percent. In Fig. 6, the return loss for the near-optimum matching section has been plotted to be compared with the performance of the optimum Dolph–Chebyshev Taper.

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Propagation Characteristics of a Rectangular Waveguide Containing a Cylindrical Rod of Magnetized Ferrite

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***Abstract*—With an axially magnetized cylindrical ferrite rod inserted into a rectangular waveguide parallel to the *E*-field of the dominant (TE_{10}) mode, the electromagnetic field amplitudes inside the ferrite rod and the transmission and reflection coefficients are numerically obtained by means of a digital computer and their results are shown in figures. At resonance, the distributions of RF magnetization and electric field have good symmetrical patterns in the cross section of the rod.**

The experimental results of the transmission and reflection coefficients agree well with the theoretical values.

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I. Introduction

The resonance phenomena of ferrite samples have been investigated by a number of authors. Among these investigations, the problems concerning propagation characteristics and field distributions in the waveguide containing a ferrite sample are of interest in view of both experiments and theories, because they are important to the studies of the nonreciprocal devices and the conversion from electromagnetic waves to magnetic waves [1]. However, the field distribution inside the ferrite sample and the detailed transmission and reflection coefficients have not been reported.