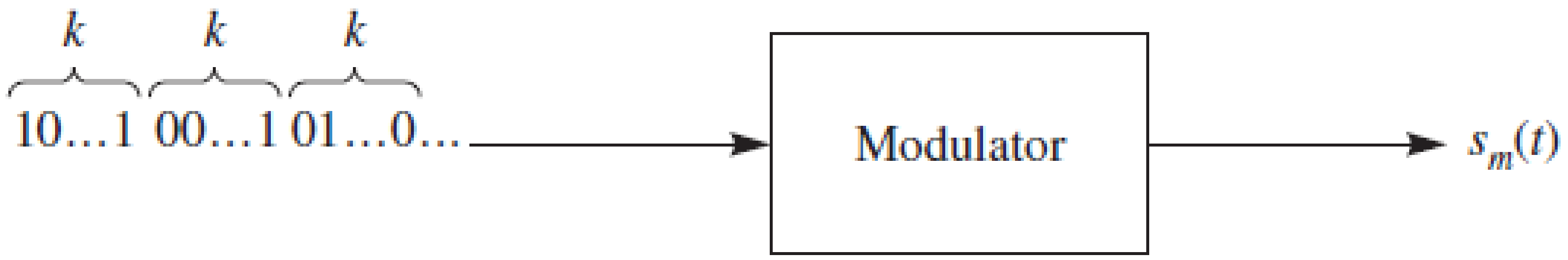
# Coherent Optical Systems

Οπτικά Δίκτυα Επικοινωνιών

Photonics Communications Research Laboratory

# Digital Modulation

- Digital Modulation is the mapping of a digital (usually binary) sequence into a set of analog signals.
- A block of $k$ bits is mapped into one of $M = 2^k$ signals $s_m(t)$ where $1 \leq m \leq 2^k$. $M$ is the modulation order.
- In this way it is possible to transmit $k$ bits of information every $T_s$ sec and as a result to increase the transmitted bit rate.



# Digital Modulation

- Different modulation schemes ($s_m(t)$) can be used:
  - PAM → Pulse Amplitude Modulation
  - PSK → Phase Shift Keying
  - QAM → Quadrature Amplitude Modulation

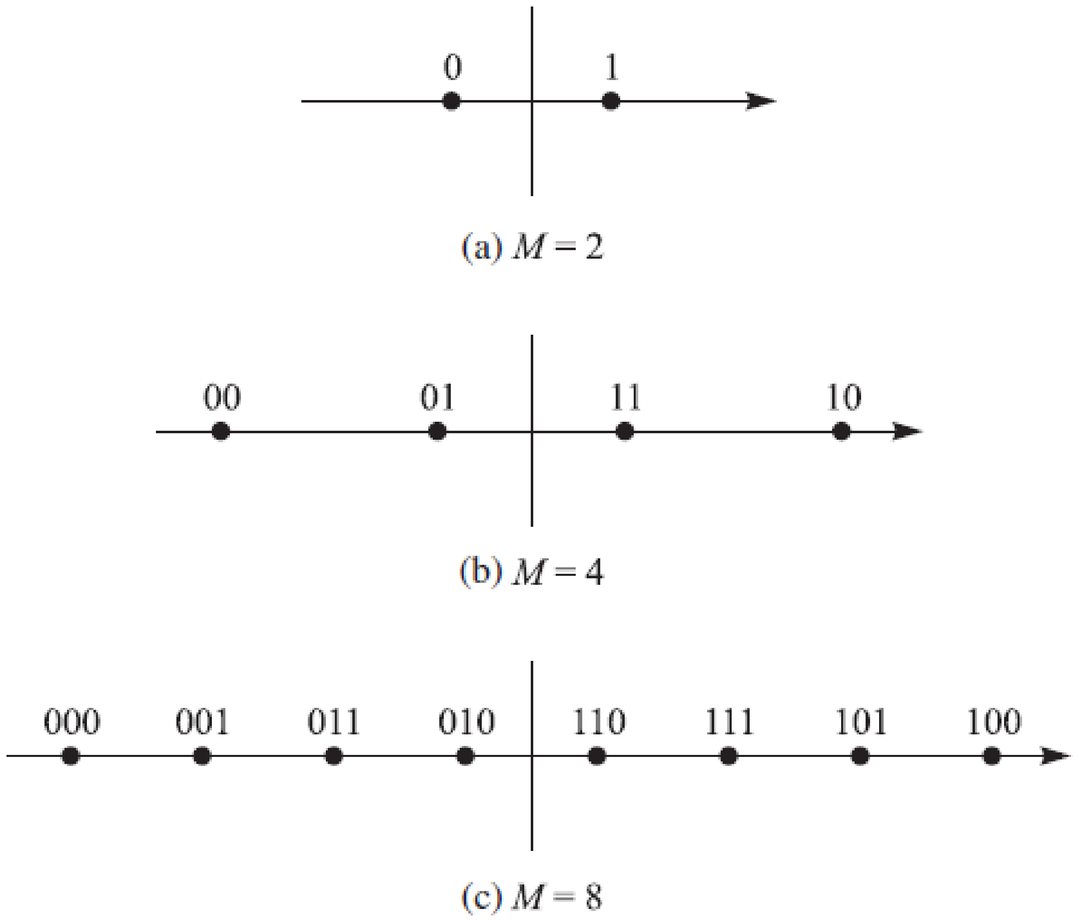
# Digital Modulation Formats

- PAM: Pulse Amplitude Modulation

$$s_m(t) = A_m \cos(2\pi f_c t)$$

$$A_m = 0,1, \dots (M-1)$$

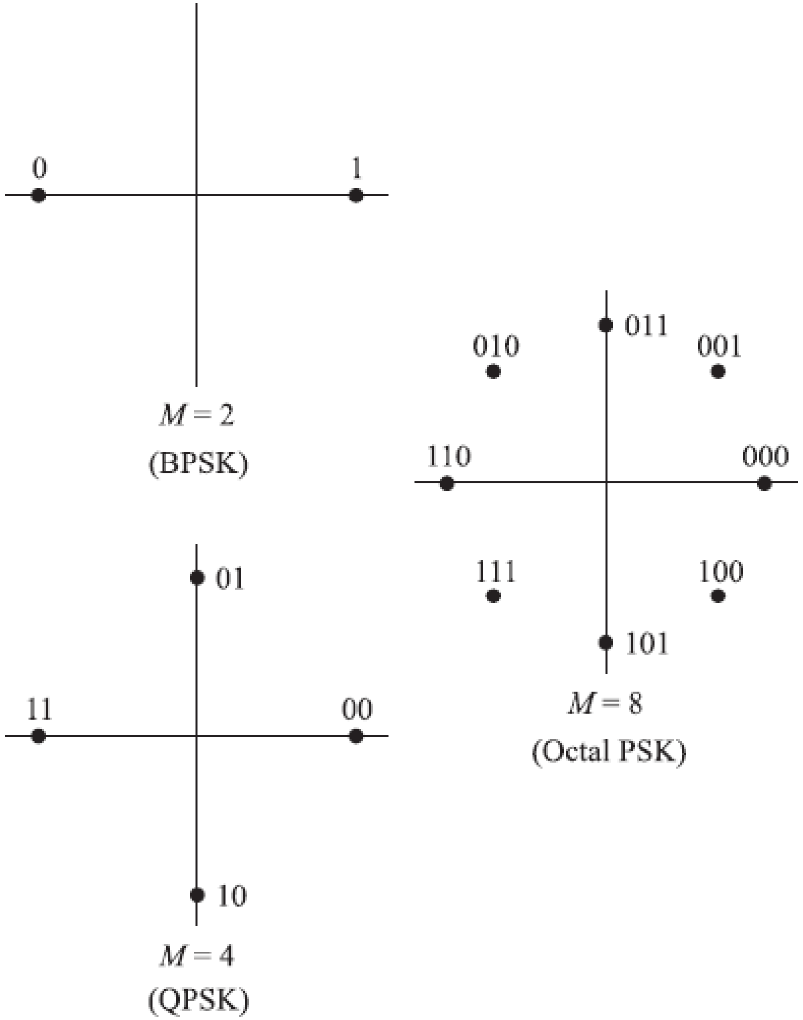
$M = 2$: On Off Keying (OOK)



# Digital Modulation Formats

- PSK: Phase Shift Keying

$$s_m(t) = g(t)\cos\left(2\pi f_c t + \frac{2\pi}{M}(m-1)\right)$$

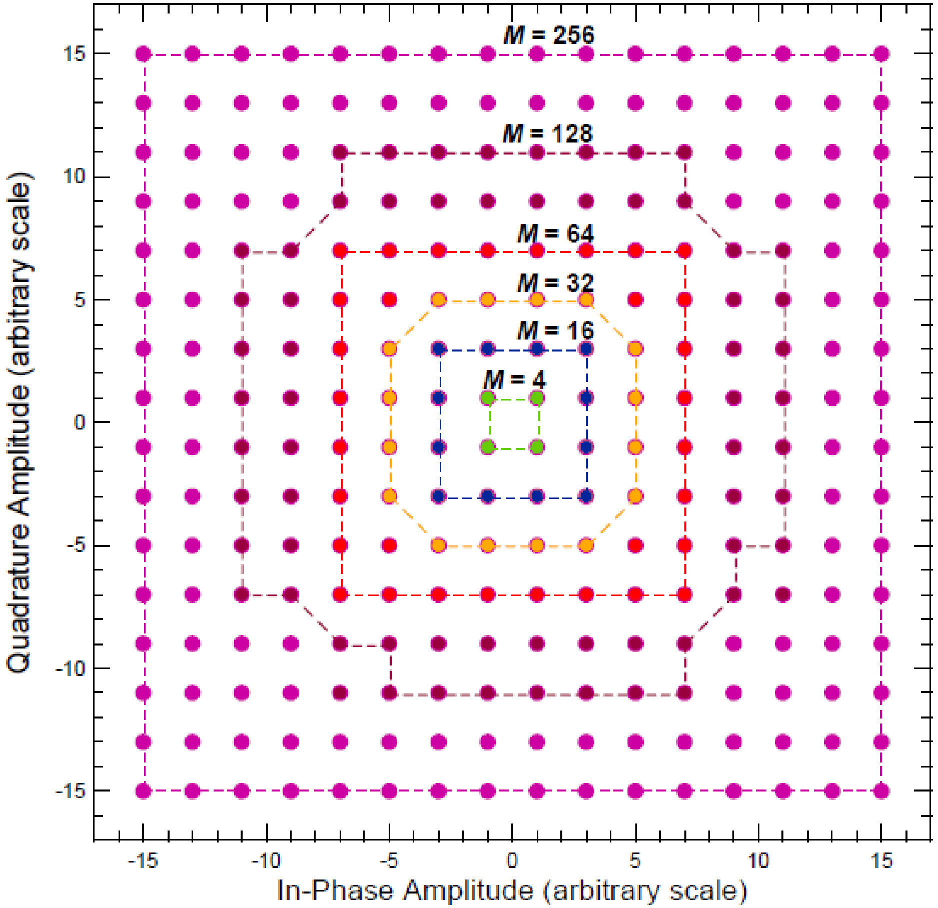


# Digital Modulation Formats

- QAM: Quadrature Amplitude Modulation

$$s_m(t) = I_m\, g(t)\cos(2\pi f_c t) - \cdots$$
$$Q_m\, g(t)\sin(2\pi f_c t)$$

$$I_m, Q_m = \pm 1, \pm 3, \ldots, (\pm M - 1)$$



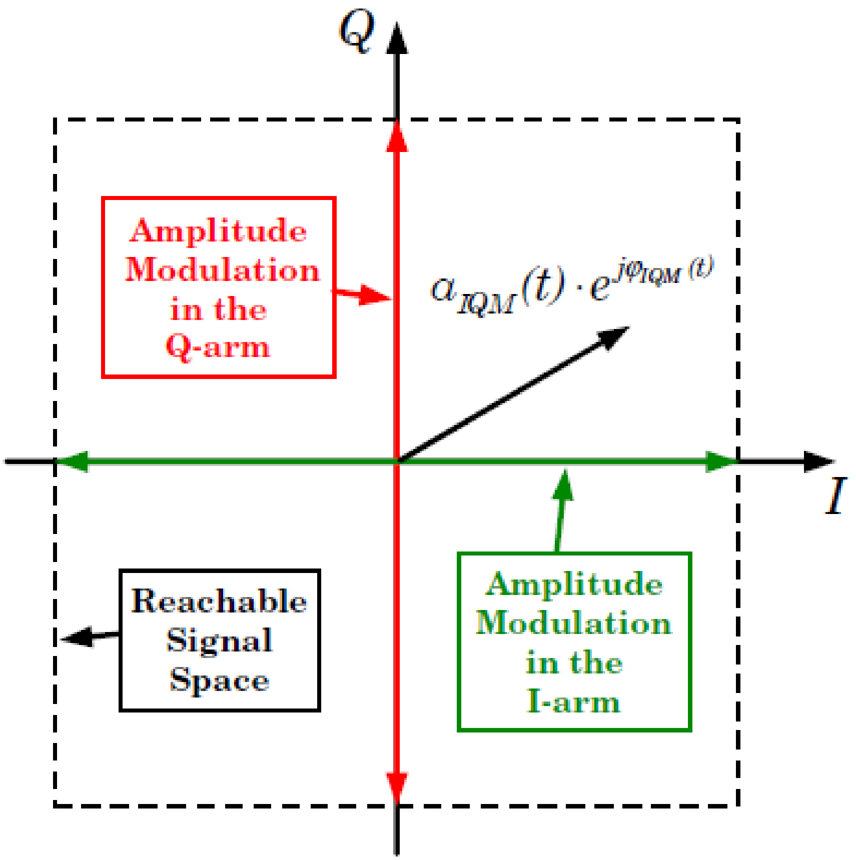
# Digital Modulation Formats

- QAM alternative representation

$$s_m(t) = Re[r_m(t)\exp(j\varphi_m(t))]$$

$$r_m(t) = g(t)\sqrt{I_m^2 + Q_m^2}$$

$$\varphi_m(t) = \tan^{-1}\left(\frac{Q_m}{I_m}\right)$$



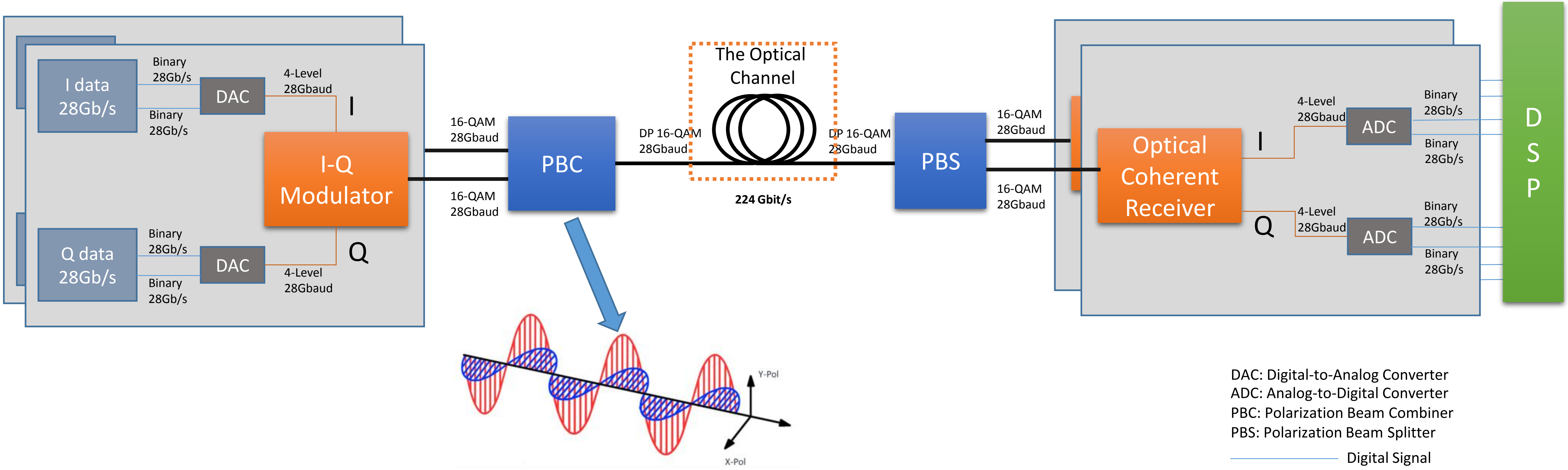
# Quadrature Amplitude Modulation (QAM)

- **Quadrature Amplitude Modulation (QAM)** is the most common advanced modulation scheme
- Depending on the multiplicity of the digital signals, $m$, the resulting QAM will have $2^m$ possible states

| bits/symbol | QAM |
|---|---|
| m=2 | QPSK |
| m=4 | 16 QAM |
| m=6 | 64 QAM |
| m=8 | 256 QAM |

# Optical Transmission Channel

## Dual Polarization 16-QAM Coherent Transmission Link

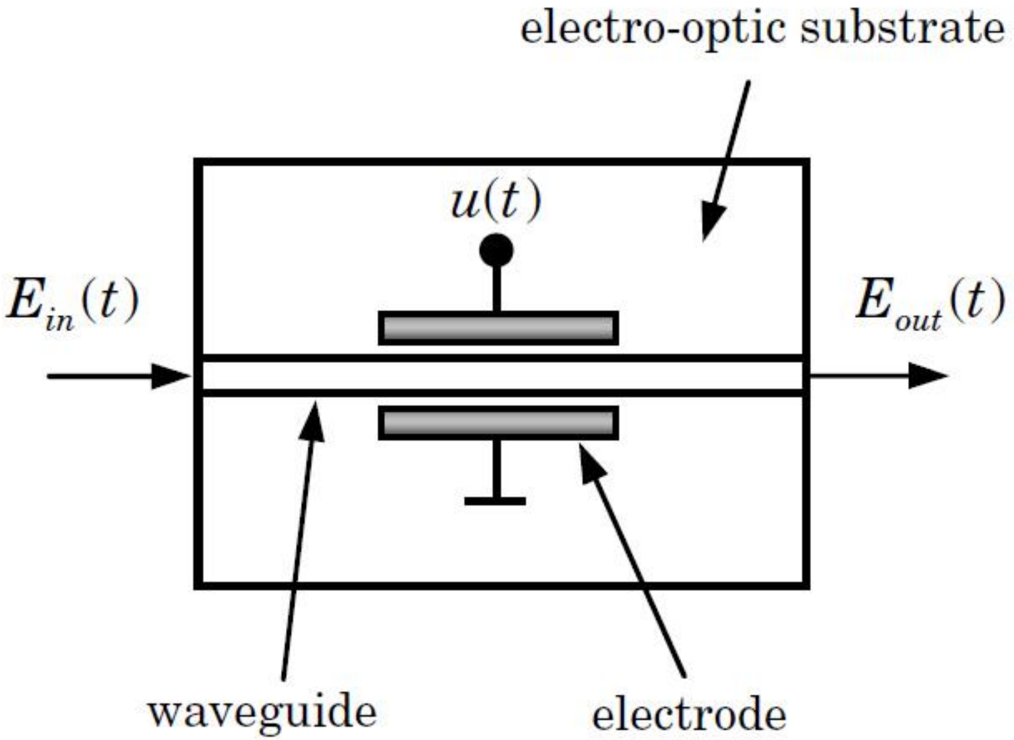



We can also modulate **two orthogonal polarizations**,
**doubling** the spectral efficiency (e.g. DP-16-QAM 8 bits/symbol)

DAC: Digital-to-Analog Converter
ADC: Analog-to-Digital Converter
PBC: Polarization Beam Combiner
PBS: Polarization Beam Splitter

Digital Signal
Analog Signal
Optical Fiber

# Phase Modulation




- Phase modulation depends on the wavelength $\boldsymbol{\lambda}$, electrode length (interaction length) $\boldsymbol{l_{el}}$, and the change of the effective refractive index $\Delta n_{eff}$.
- Considering only the Pockels effect, the change of the refractive index can be assumed to be linear w.r.t. the applied external voltage *u(t)*:

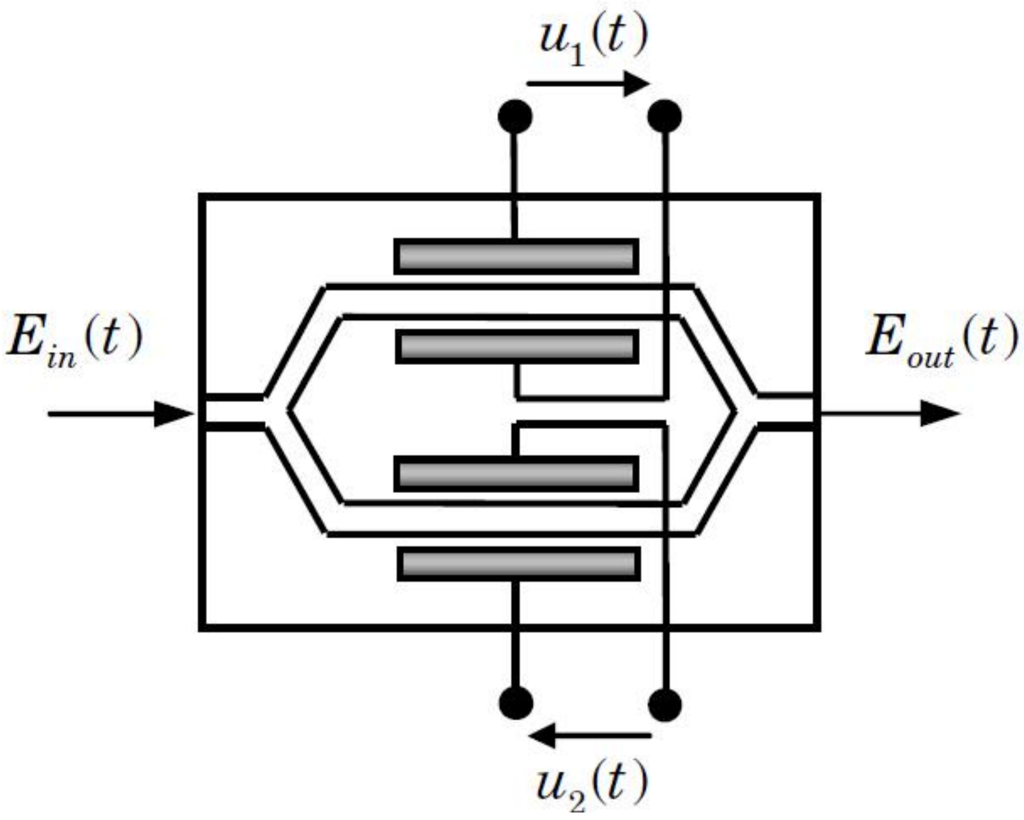
$$\varphi_{PM}(t) = \frac{2\pi}{\lambda} \cdot \Delta n_{eff}(t) \cdot l_{el} \sim u(t)$$

- The most important specification given is $\boldsymbol{V_\pi}$: The voltage required to produce a phase shift of $\boldsymbol{\pi}$.

## Transfer Function:

$$\boxed{E_{out}(t) = E_{in}(t) \cdot e^{j\varphi_{PM}(t)} = E_{in}(t) \cdot e^{j\frac{u(t)}{V_\pi}\pi}}$$

# Mach-Zehnder Modulator (MZM)



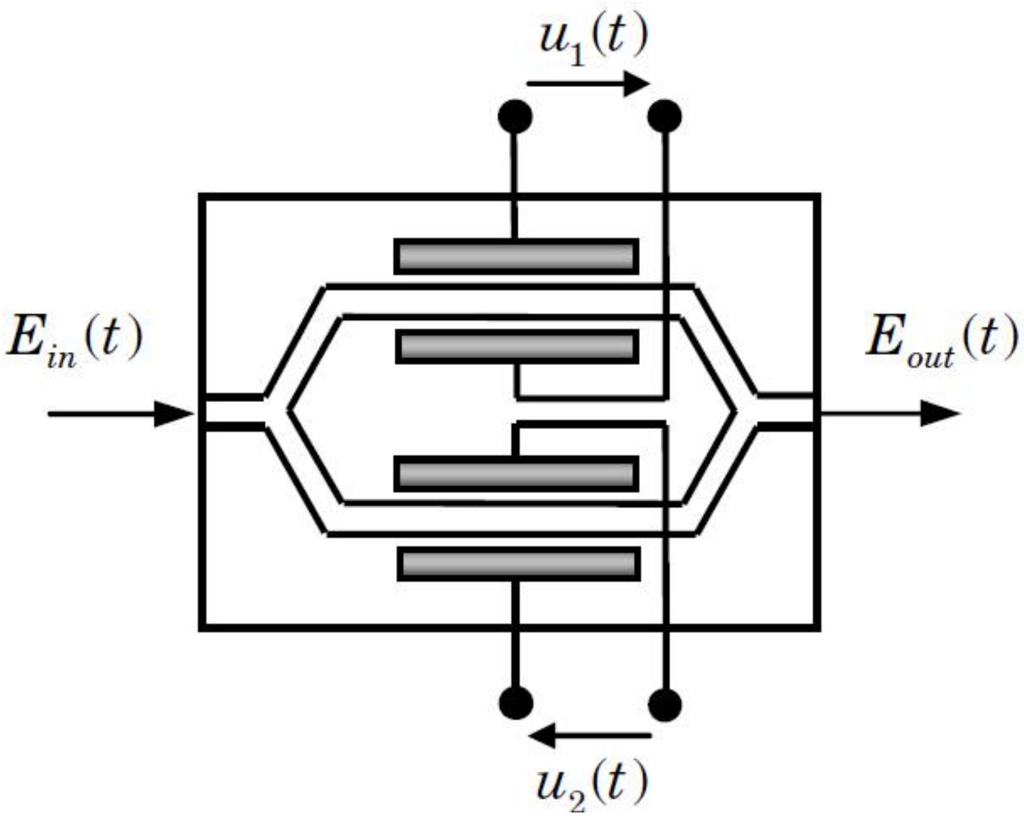

- Two phase modulators can be placed in parallel using an interferometric structure.
- The incoming light is split into two branches, different phase shifts applies to each path, and then recombined.
- The output is a result of interference, ranging from constructive (the phase of the light in each branch is the same) to destructive (the phase in each branch differs by ***π***).

## Transfer Function:

$$\boxed{\frac{E_{out}(t)}{E_{in}(t)} = \frac{1}{2} \cdot \left(e^{j\varphi_1(t)} + e^{j\varphi_2(t)}\right)}$$

**where:** $\varphi_1(t) = \frac{u_1(t)}{V_{\pi_1}}\pi, \; \varphi_2(t) = \frac{u_2(t)}{V_{\pi_2}}\pi$

# Mach-Zehnder Modulator (MZM)




- **Push-Push Operation**
  - ***u1(t)*** = ***u2(t)***
  - Pure phase modulation
- **Push-Pull Operation**
  - ***u1(t)*** = -***u2(t)***
  - Pure amplitude modulation

## Transfer Function:

$$\frac{E_{out}(t)}{E_{in}(t)} = \frac{1}{2} \cdot \left( e^{j\varphi_1(t)} + e^{j\varphi_2(t)} \right)$$

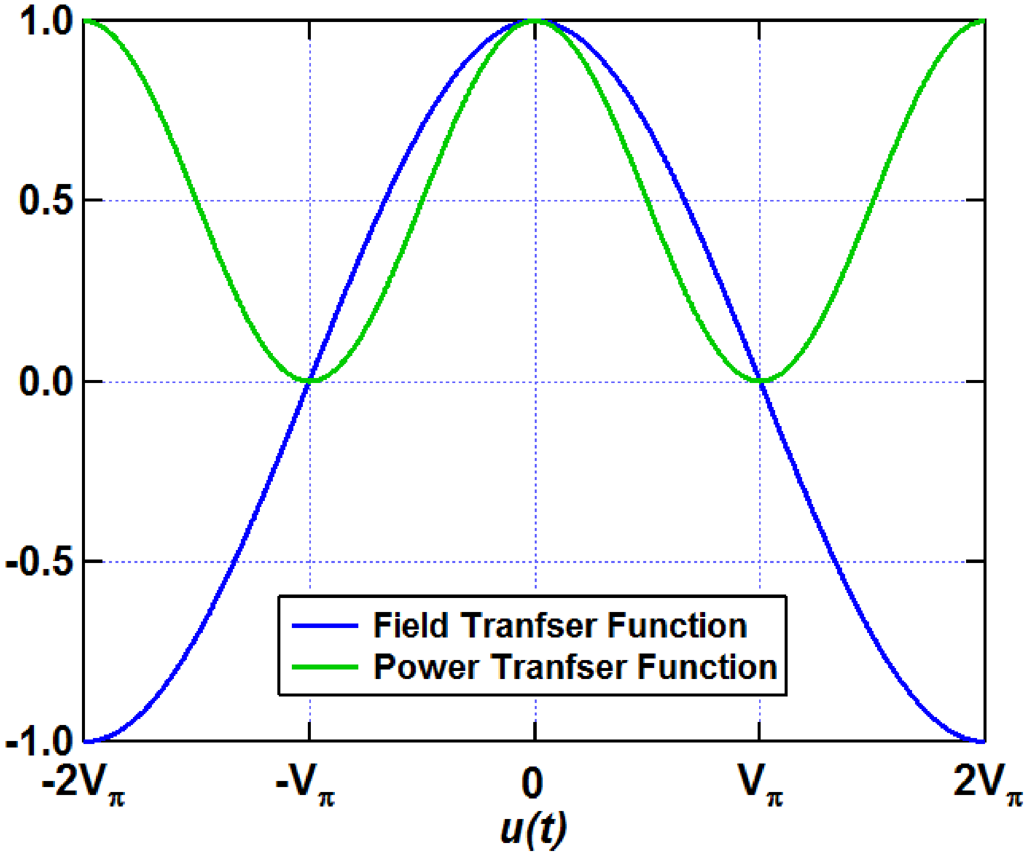
**where:** $\varphi_1(t) = \frac{u_1(t)}{V_{\pi_1}}\pi, \ \varphi_2(t) = \frac{u_2(t)}{V_{\pi_2}}\pi$

# Mach-Zehnder Modulator (MZM)

- When operating in **Push-Pull** with ***u1(t)*** = -***u2(t)*** = ***u(t)/2***, the **field** (***E***) and **power** (***P***) transfer functions are (the power T.F. is obtained by squaring the field T.F.):

$$E_{out}(t) = E_{in}(t) \cdot \cos\left(\frac{\Delta\varphi_{MZM}(t)}{2}\right) = E_{in}(t) \cdot \cos\left(\frac{u(t)}{2V_\pi}\pi\right)$$

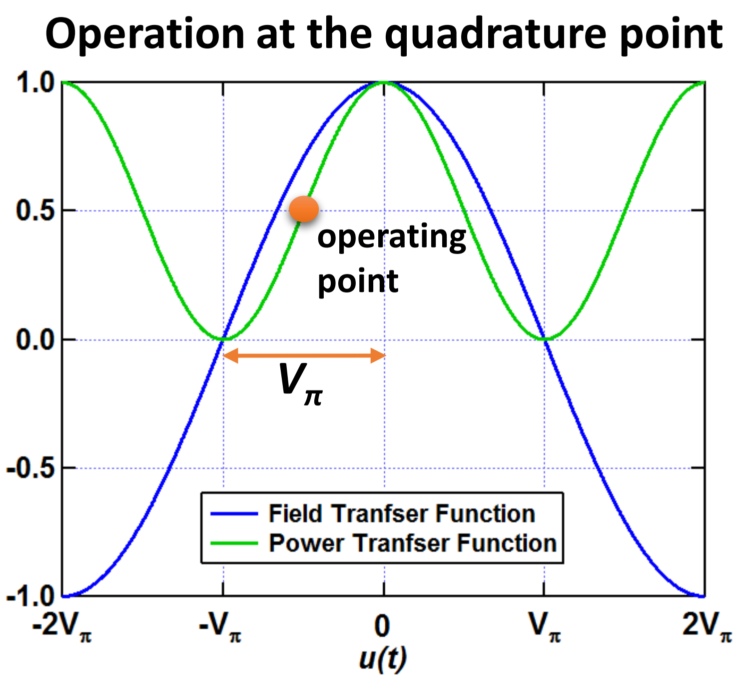
$$\frac{P_{out}(t)}{P_{in}(t)} = \frac{1}{2} + \frac{1}{2} \cdot \cos\left(\Delta\varphi_{MZM}(t)\right) = \frac{1}{2} + \frac{1}{2} \cdot \cos\left(\frac{u(t)}{V_\pi}\pi\right)$$

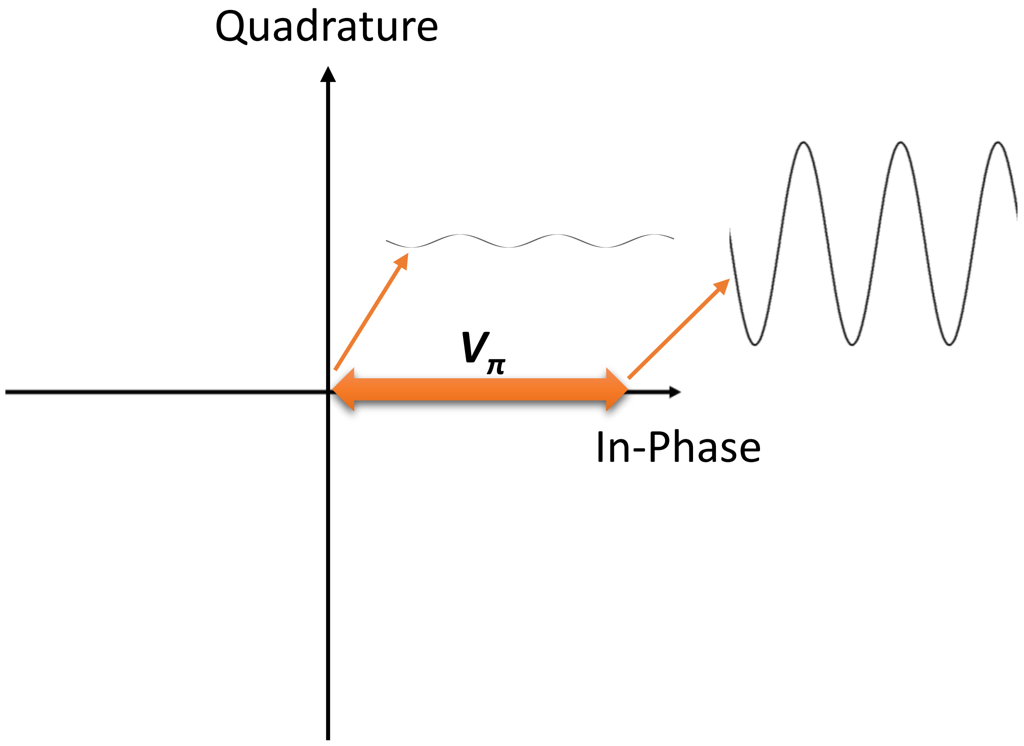


# Mach-Zehnder Modulator (MZM)

- Set bias voltage at **quadrature point** ($V_{bias} = -V_\pi/2$) and modulate with an input voltage swing of $\boldsymbol{V_\pi}$ peak-to-peak.

  → Pure amplitude modulation (**On-Off Keying**)

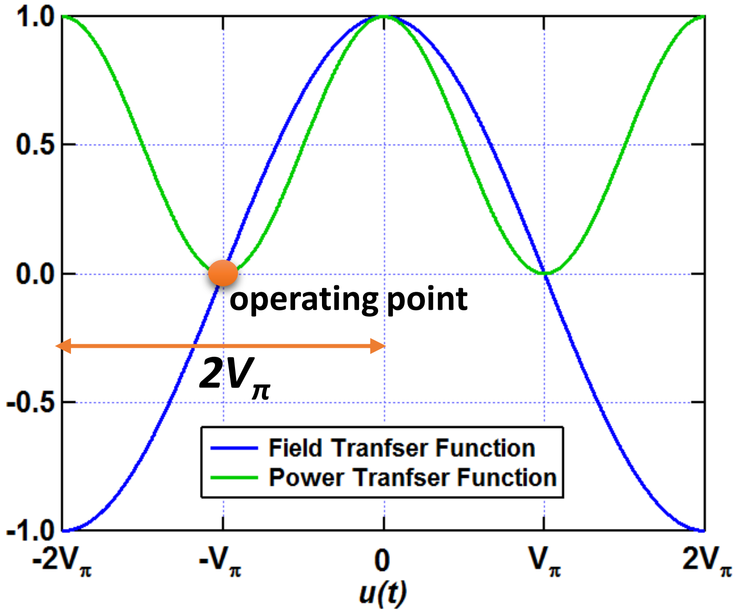


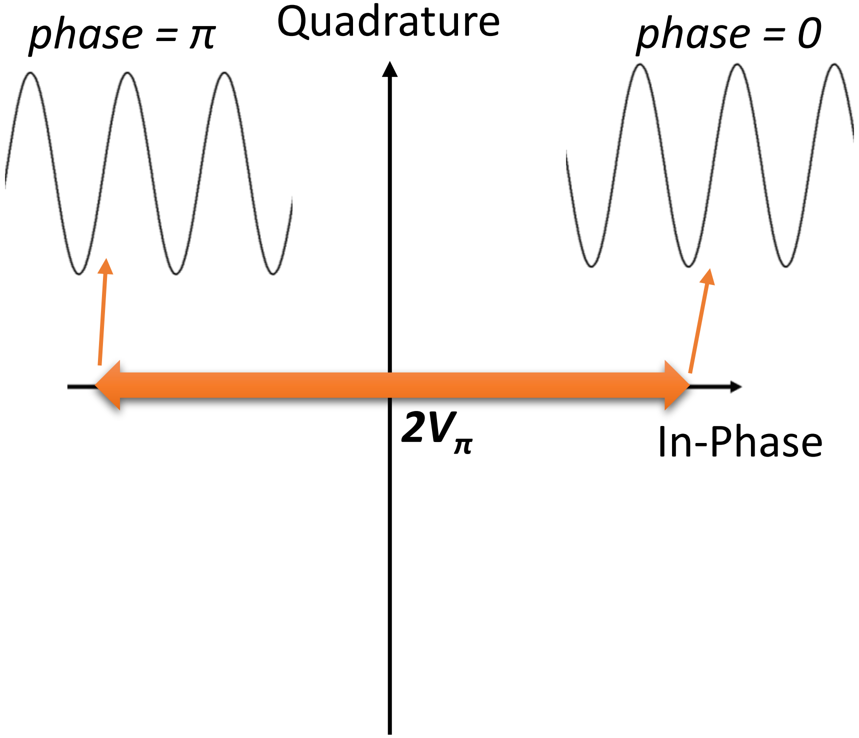



# Mach-Zehnder Modulator (MZM)

- Set bias voltage at **minimum transmission point** ($V_{bias}$ = -$V_\pi$) and modulate with an input voltage swing of **2·$V_\pi$** peak-to-peak.
  - → In addition to amplitude modulation, a phase skip of $\boldsymbol{\pi}$ occurs every time the input, ***u(t)***, crosses the minimum transmission point (example: **BPSK**).

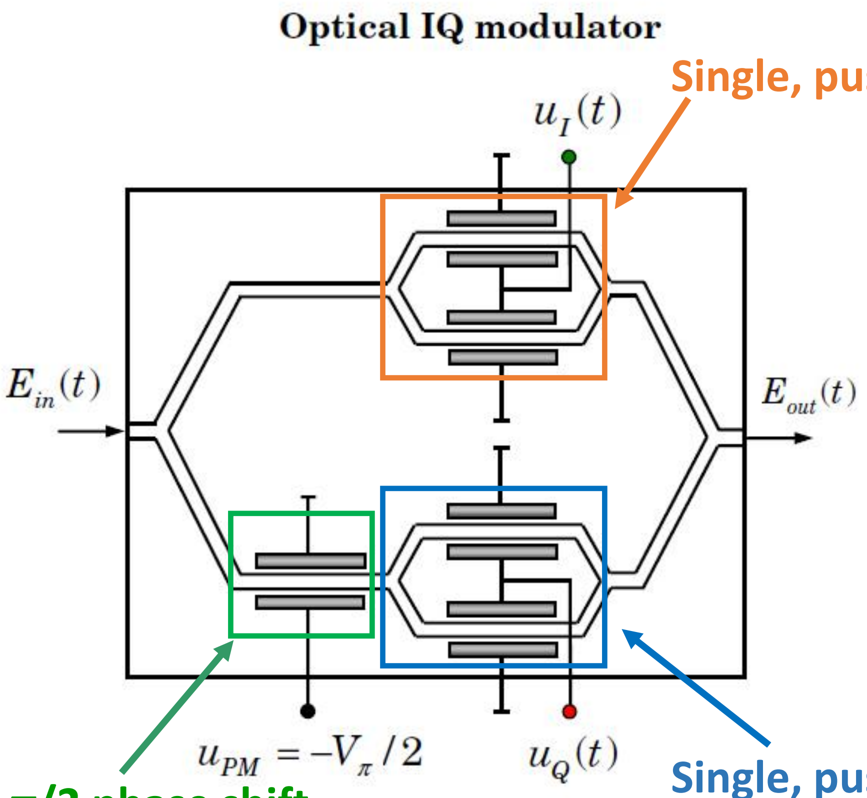
**Operation at the minimum transmission point**



# Optical IQ Modulator

- Dual-Nested IQ (In-Phase, Quadrature) Mach-Zehnder Modulator (with each MZM biased at minimum transmission point).




Single, push-pull MZM modulating the <u>In-Phase</u> component

π/2 phase shift

Single, push-pull MZM modulating the <u>Quadrature</u> component

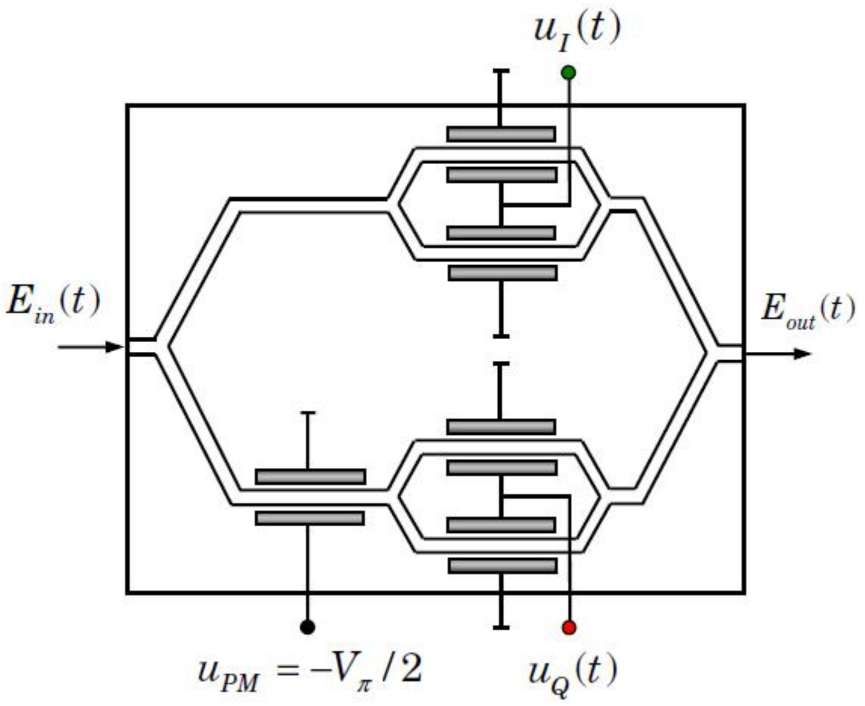
$$\frac{E_{out}(t)}{E_{in}(t)} = \boxed{\frac{1}{2}\cos\left(\frac{\Delta\varphi_I(t)}{2}\right)} + \boxed{j}\boxed{\frac{1}{2}\cos\left(\frac{\Delta\varphi_Q(t)}{2}\right)}$$

$$\Delta\varphi_I(t) = \frac{u_I(t)}{V_\pi}\pi,\ \Delta\varphi_Q(t) = \frac{u_Q(t)}{V_\pi}\pi.$$

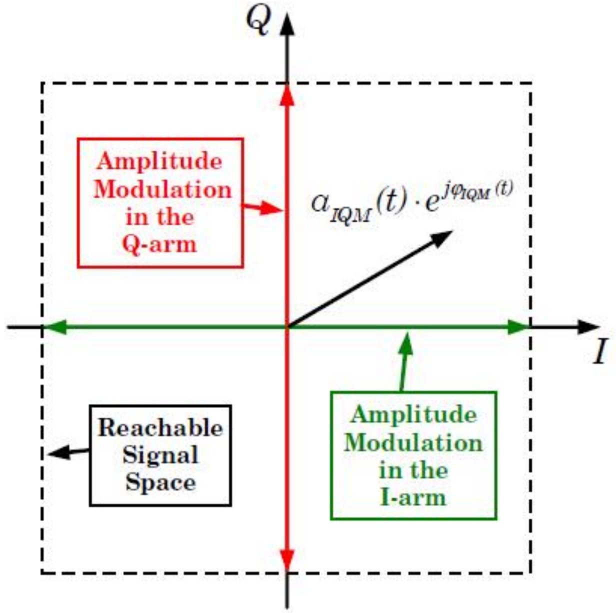
# Optical IQ Modulator

- Dual-Nested IQ (In-Phase, Quadrature) Mach-Zehnder Modulator (with each MZM biased at minimum transmission point).

**Optical IQ modulator**




**Principle of IQ modulation**

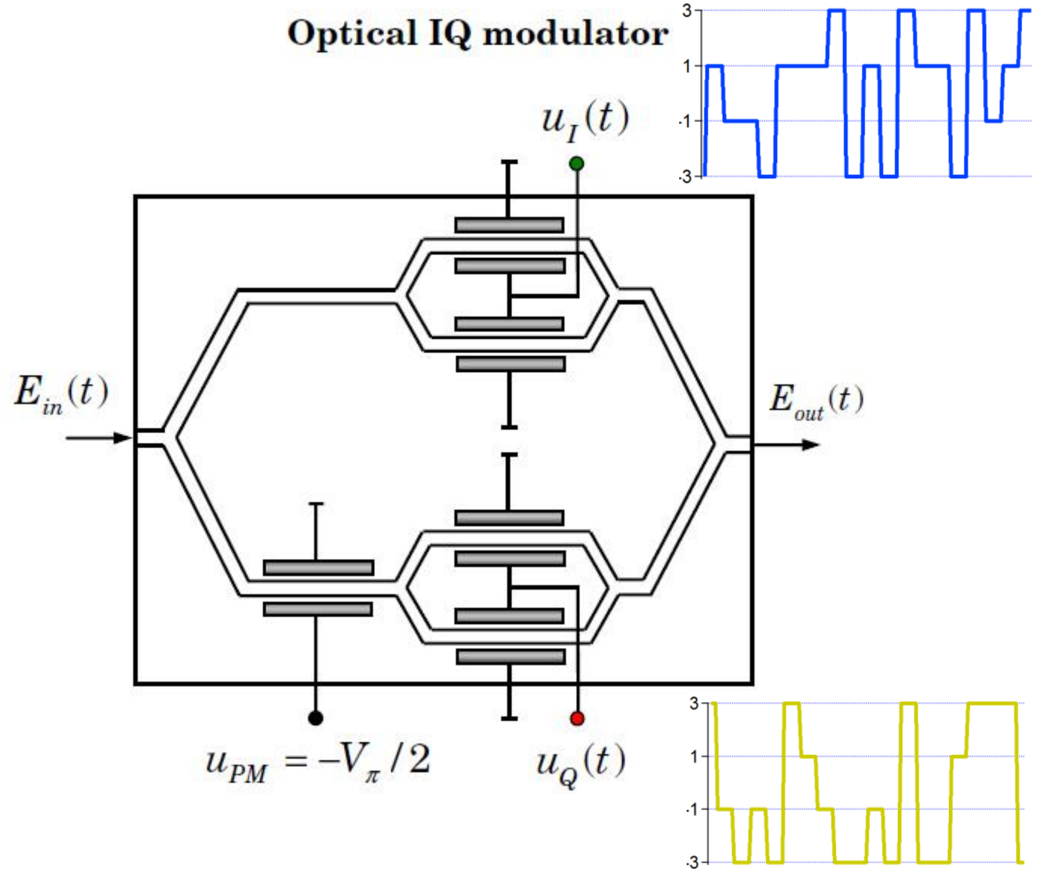


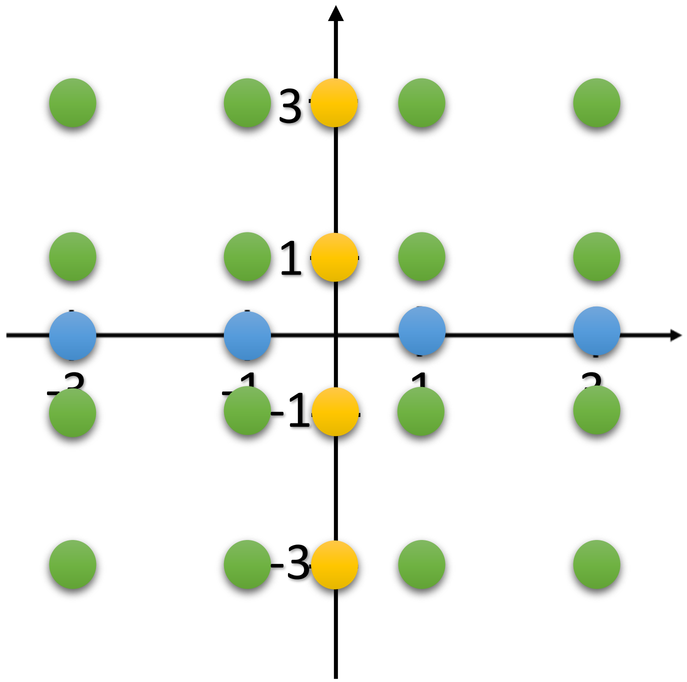

$$\frac{E_{out}(t)}{E_{in}(t)} = \frac{1}{2}\cos\left(\frac{\Delta\varphi_I(t)}{2}\right) + j\,\frac{1}{2}\cos\left(\frac{\Delta\varphi_Q(t)}{2}\right)$$

$$\Delta\varphi_I(t) = \frac{u_I(t)}{V_\pi}\pi, \quad \Delta\varphi_Q(t) = \frac{u_Q(t)}{V_\pi}\pi.$$

# Optical IQ Modulator

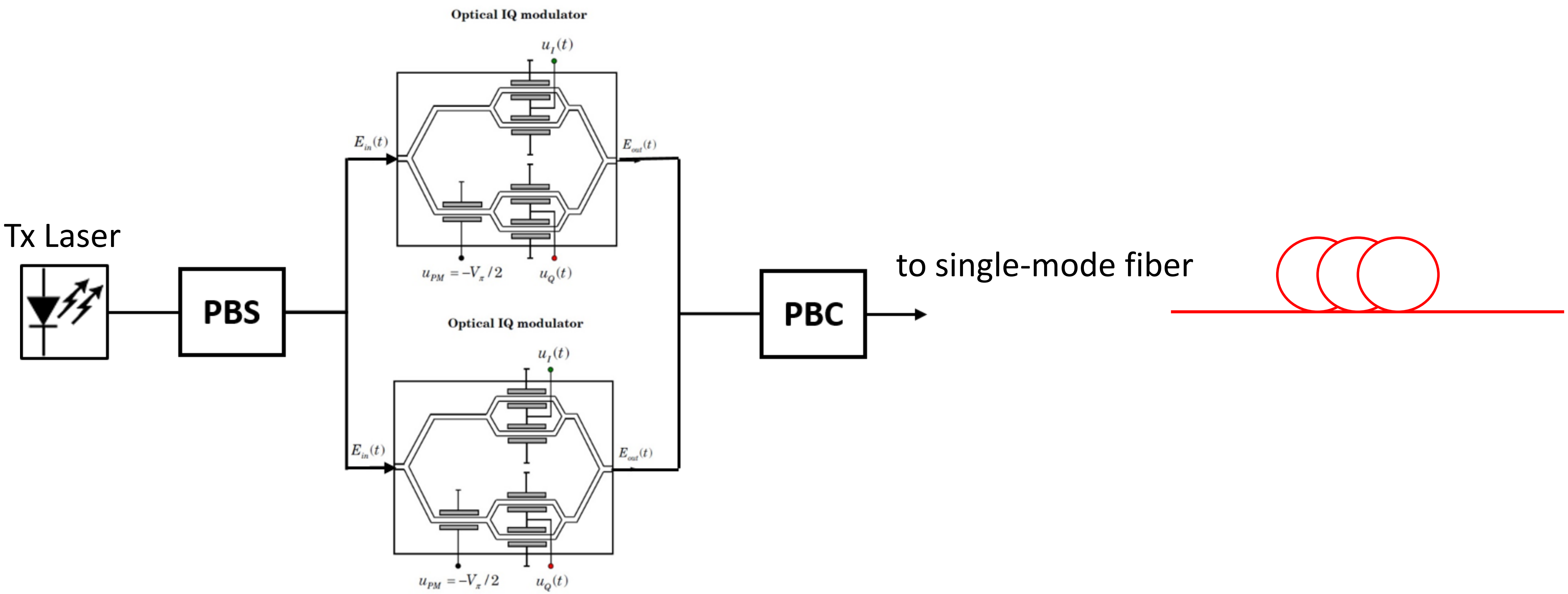
- <u>Example:</u> If we apply 4-level Pulse Amplitude Modulation (4-PAM) signals on each MZM (***u**$_I$* and ***u**$_Q$*), the resulting output of the IQ modulator is 16-QAM.



# Dual Polarization Transmitter

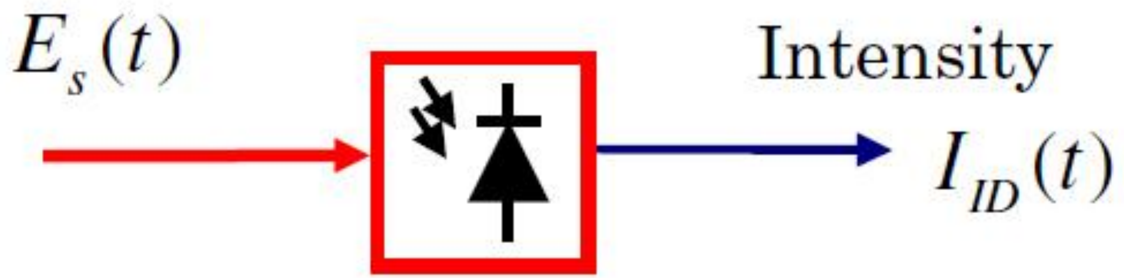
- A **Polarization Beam Splitter** (**PBS**) is used to split the light from the Tx laser into two orthogonal polarizations, each of which is modulated by an IQ-MZM.
- A **Polarization Beam Combiner** (**PBC**) recombines the two signals at the output.



# Direct Detection

- In **intensity modulated formats** the digital information is recovered through **direct detection** at the optical receiver, through a **photodiode** that converts the power of the optical carrier into electrical current. The photocurrent at the output of the photodiode is proportional to the square of the signal amplitude:

$$I_{ID} \propto E_S \cdot E_S' = \left(A_S e^{j(\omega_S t+\varphi)}\right) \cdot \left(A_S e^{-j(\omega_S t+\varphi)}\right) = A_S^2$$

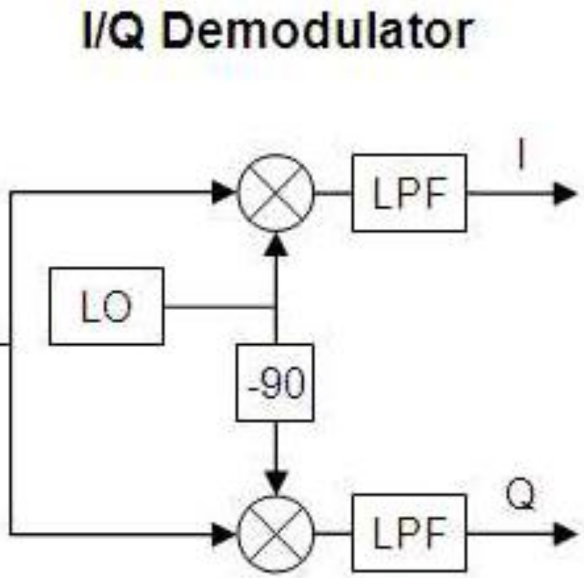



- This results in **loss of phase information**, and is therefore unsuitable for advanced modulation formats that use the phase dimension to encode data information.

# IQ Demodulation

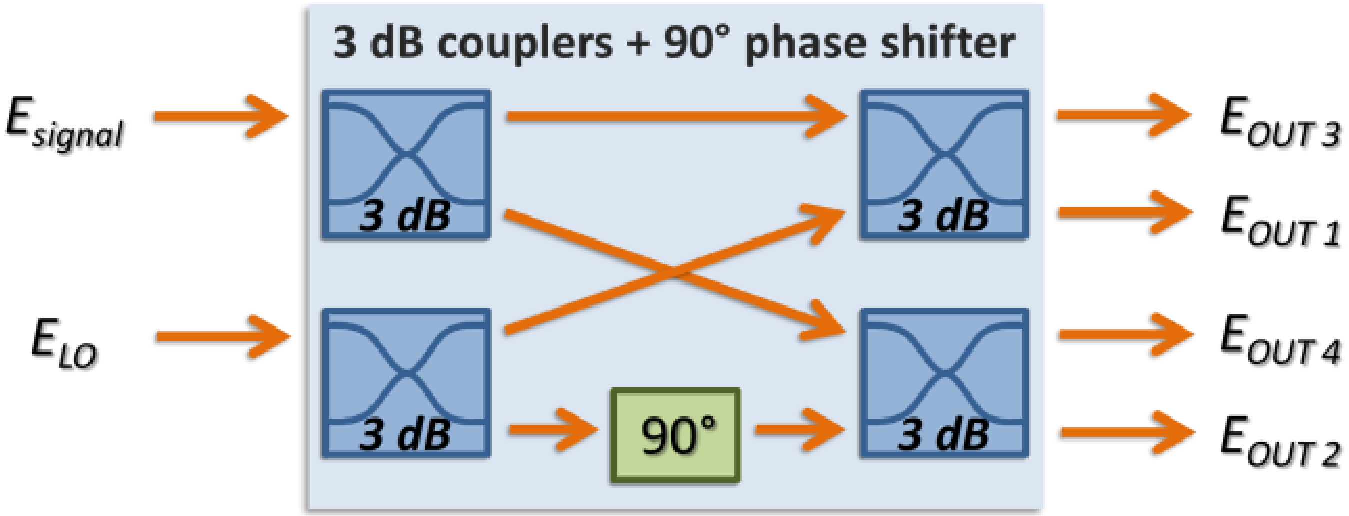
- An IQ demodulator mixes the received modulated carrier with a Continuous Wave (CW) Local Oscillator (LO), and a 90-degree shifted version of the LO - i.e. with **cos(2$\pi f_{LO}$)** and **-sin(2$\pi f_{LO}$)**
- If $f_{LO} = f_c$, the signal is **downconverted from the carrier frequency down to baseband**, and the **in-phase** and **quadrature** components can be recovered.
- Its functionality is to essentially obtain the **complex envelope** (and therefore, the data) of a modulated carrier.

Received modulated carrier (frequency = $f_c$)
$I(t) \cdot \cos(2\pi f_c t) + Q(t) \cdot \sin(2\pi f_c t)$



# 90 degree hybrid

- In optical communications, the IQ demodulator is called the **90 degree hybrid**. It is used to beat (mix) the signal ($\boldsymbol{E_s}$) with the LO ($\boldsymbol{E_{lo}}$), as well as the 90 degree shifted version of the LO.



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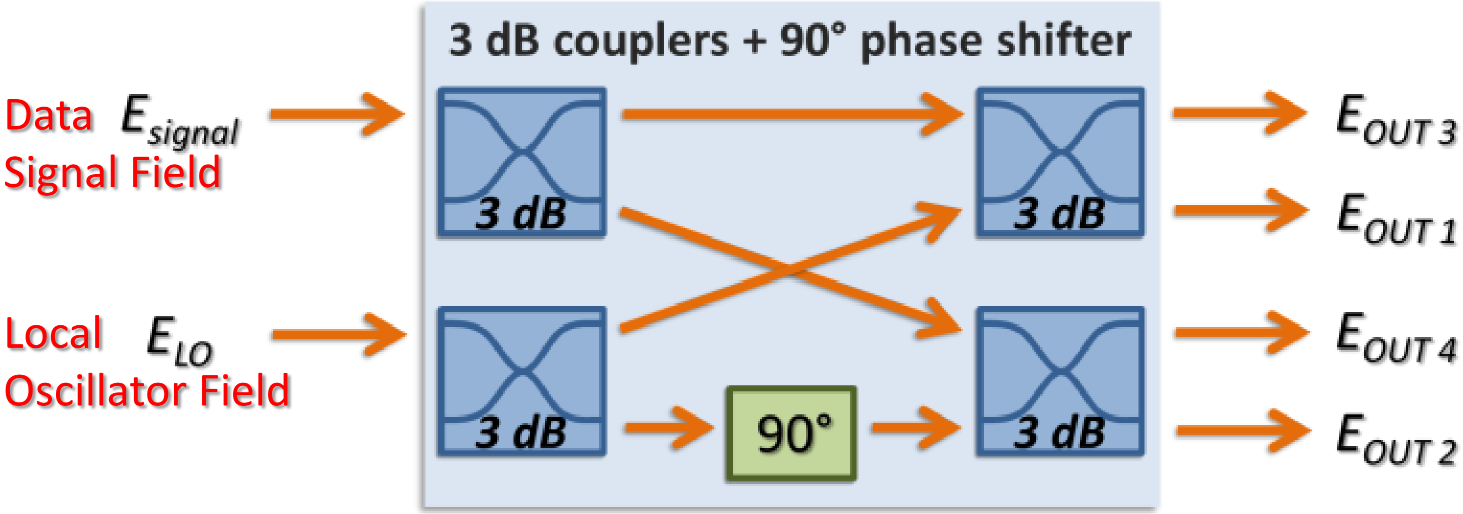
Power, frequency and phase of optical carrier

Modulation amplitude and phase (QAM data!)

$$\mathbf{E}_s(t) = \sqrt{P_s} \cdot e^{j(\omega_s t+\varphi_s)} \cdot a(t) \cdot e^{j\varphi(t)} \cdot e^{j\varphi_{n_s}(t)}$$

Phase noise of Tx laser

$$\mathbf{E}_{lo}(t) = \sqrt{P_{lo}} \cdot e^{j(\omega_{lo} t+\varphi_{lo})} \cdot e^{j\varphi_{n_{lo}}(t)}$$



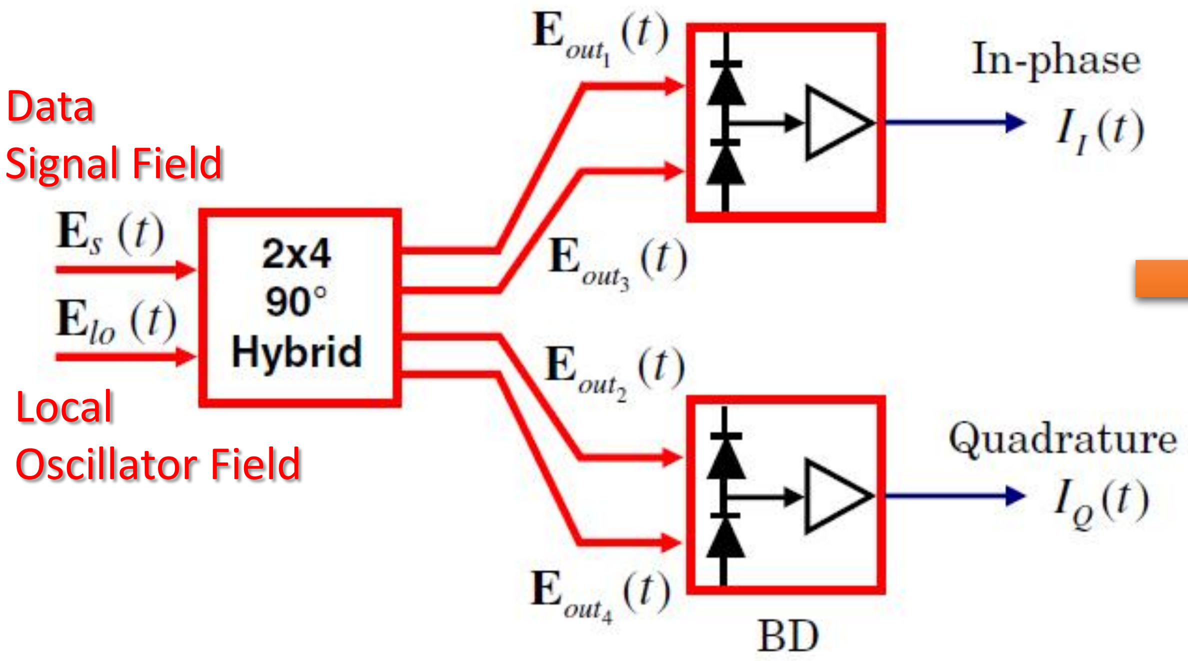

$$\begin{bmatrix} \mathbf{E}_{out_1}(t) \\ \mathbf{E}_{out_2}(t) \\ \mathbf{E}_{out_3}(t) \\ \mathbf{E}_{out_4}(t) \end{bmatrix} = \frac{1}{2} \cdot \begin{bmatrix} \mathbf{E}_s(t) + \mathbf{E}_{lo}(t) \\ \mathbf{E}_s(t) + j\mathbf{E}_{lo}(t) \\ \mathbf{E}_s(t) - \mathbf{E}_{lo}(t) \\ \mathbf{E}_s(t) - j\mathbf{E}_{lo}(t) \end{bmatrix}$$

**Signal + LO**

**Signal + 90$^{o}$ shifted LO**

# Coherent Detection

- After detection of the outputs in balanced photodiodes the **in-phase** and **quadrature** components of the data signal (referenced to the CW local oscillator) are recovered.



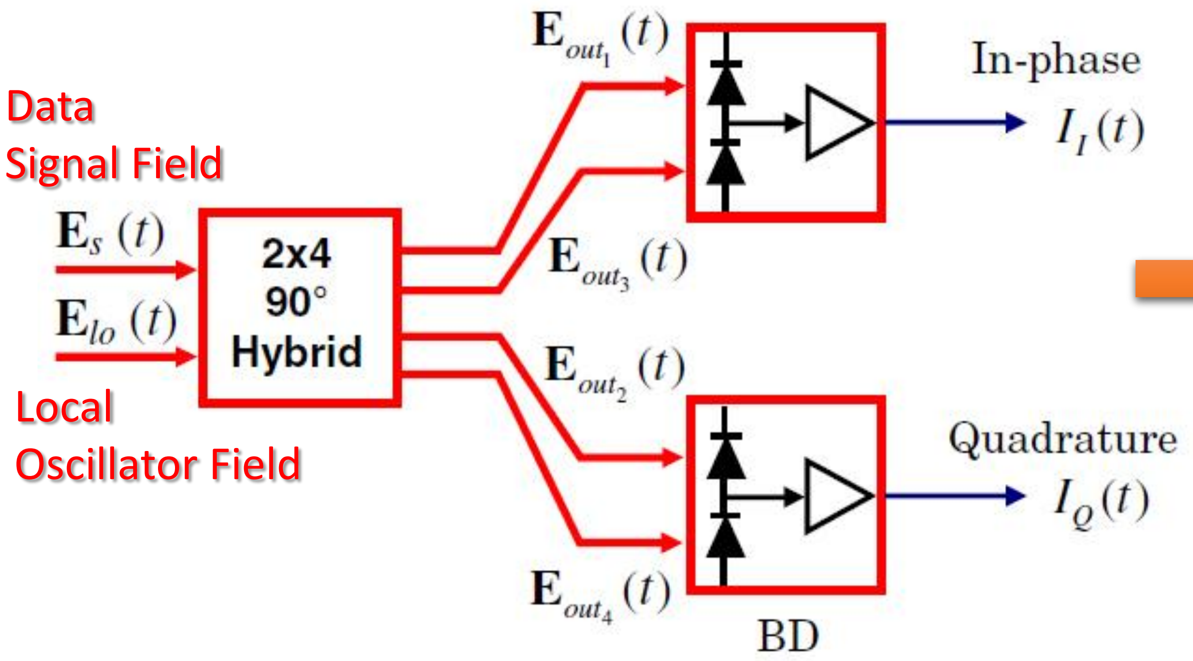

$$\begin{bmatrix} \mathbf{E}_{out_1}(t) \\ \mathbf{E}_{out_2}(t) \\ \mathbf{E}_{out_3}(t) \\ \mathbf{E}_{out_4}(t) \end{bmatrix} = \frac{1}{2} \cdot \begin{bmatrix} \mathbf{E}_s(t) + \mathbf{E}_{lo}(t) \\ \mathbf{E}_s(t) + j\mathbf{E}_{lo}(t) \\ \mathbf{E}_s(t) - \mathbf{E}_{lo}(t) \\ \mathbf{E}_s(t) - j\mathbf{E}_{lo}(t) \end{bmatrix}$$

$$\begin{aligned} I_I(t) &= R \cdot \mathbf{E}_{out_1}(t) \cdot \mathbf{E}^*_{out_1}(t) - R \cdot \mathbf{E}_{out_3}(t) \cdot \mathbf{E}^*_{out_3}(t) \\ &= R\sqrt{P_s P_{lo}}\, \boxed{a(t)}\, \mathbf{e}_s \mathbf{e}_{lo} \underline{\cos} \left[\Delta\omega t + \varphi_n(t) + \varphi_0 + \boxed{\varphi(t)}\right] + i_{sh_I} \end{aligned}$$

$$\begin{aligned} I_Q(t) &= R \cdot \mathbf{E}_{out_2}(t) \cdot \mathbf{E}^*_{out_2}(t) - R \cdot \mathbf{E}_{out_4}(t) \cdot \mathbf{E}^*_{out_4}(t) \\ &= R\sqrt{P_s P_{lo}}\, \boxed{a(t)}\, \mathbf{e}_s \mathbf{e}_{lo} \underline{\sin} \left[\Delta\omega t + \varphi_n(t) + \varphi_0 + \boxed{\varphi(t)}\right] + i_{sh_Q} \end{aligned}$$

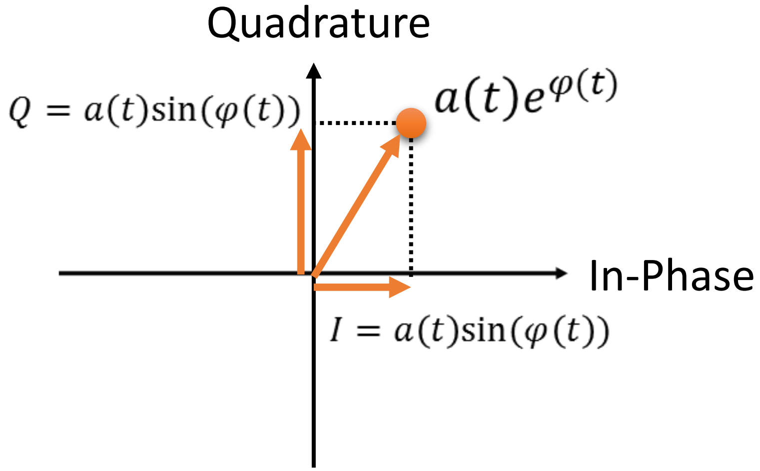
# Coherent Detection

- After detection of the outputs in balanced photodiodes the **in-phase** and **quadrature** components of the data signal (referenced to the CW local oscillator) are recovered.



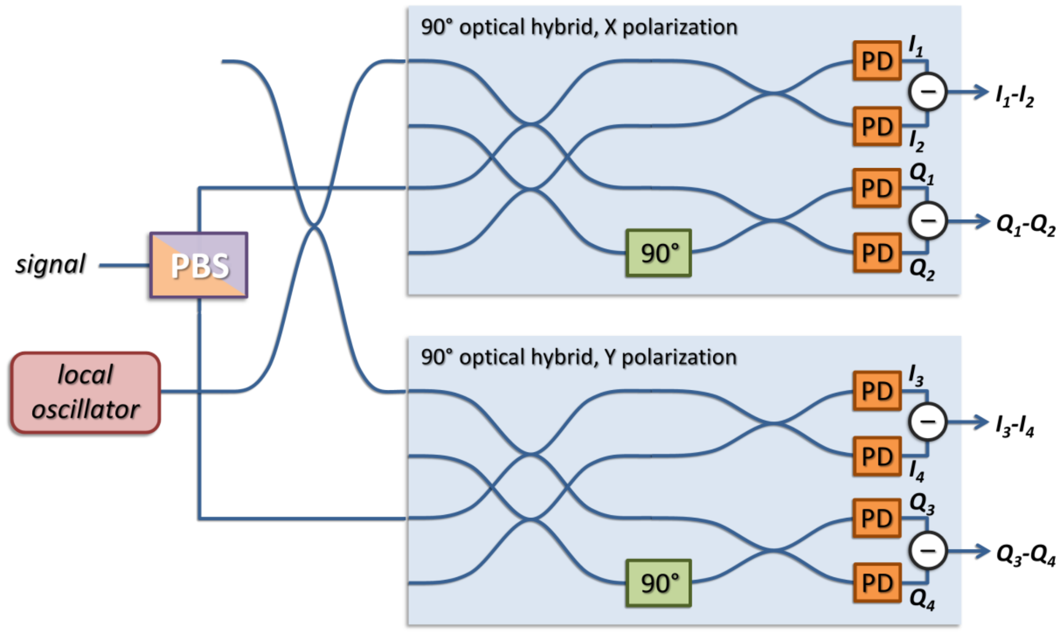

$$
\begin{aligned}
I_I(t) &= R \cdot \mathbf{E}_{out_1}(t) \cdot \mathbf{E}^*_{out_1}(t) - R \cdot \mathbf{E}_{out_3}(t) \cdot \mathbf{E}^*_{out_3}(t) \\
&= R\sqrt{P_s P_{lo}}\, \underline{a(t)\, \mathbf{e}_s \mathbf{e}_{lo} \cos\left[\Delta\omega t + \varphi_n(t) + \varphi_0 + \varphi(t)\right]} + i_{sh_I}
\end{aligned}
$$

$$
\begin{aligned}
I_Q(t) &= R \cdot \mathbf{E}_{out_2}(t) \cdot \mathbf{E}^*_{out_2}(t) - R \cdot \mathbf{E}_{out_4}(t) \cdot \mathbf{E}^*_{out_4}(t) \\
&= R\sqrt{P_s P_{lo}}\, \underline{a(t)\, \mathbf{e}_s \mathbf{e}_{lo} \sin\left[\Delta\omega t + \varphi_n(t) + \varphi_0 + \varphi(t)\right]} + i_{sh_Q}
\end{aligned}
$$



# Polarization Diversity Coherent Receiver

- Two coherent receivers are employed to detect the two orthogonal polarizations of the received signal (the polarizations are separated using a Polarization Beam Splitter).




$$I_1(t) - I_2(t) \propto 2R\sqrt{P_s^h P_{lo}} \cdot a(t) \cdot \cos[\Delta\omega t + \varphi_n(t) + \varphi_0 + \varphi(t)]$$

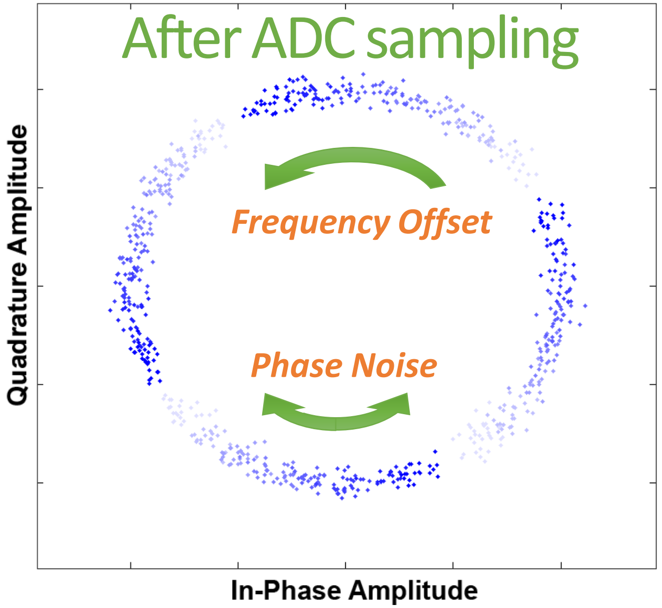
$$Q_1(t) - Q_2(t) \propto 2R\sqrt{P_s^h P_{lo}} \cdot a(t) \cdot \sin[\Delta\omega t + \varphi_n(t) + \varphi_0 + \varphi(t)]$$

$$I_3(t) - I_4(t) \propto 2R\sqrt{P_s^v P_{lo}} \cdot a(t) \cdot \cos[\Delta\omega t + \varphi_n(t) + \varphi_0 + \varphi(t)]$$

$$Q_3(t) - Q_4(t) \propto 2R\sqrt{P_s^v P_{lo}} \cdot a(t) \cdot \sin[\Delta\omega t + \varphi_n(t) + \varphi_0 + \varphi(t)]$$

# Carrier Frequency Offset and Phase Noise

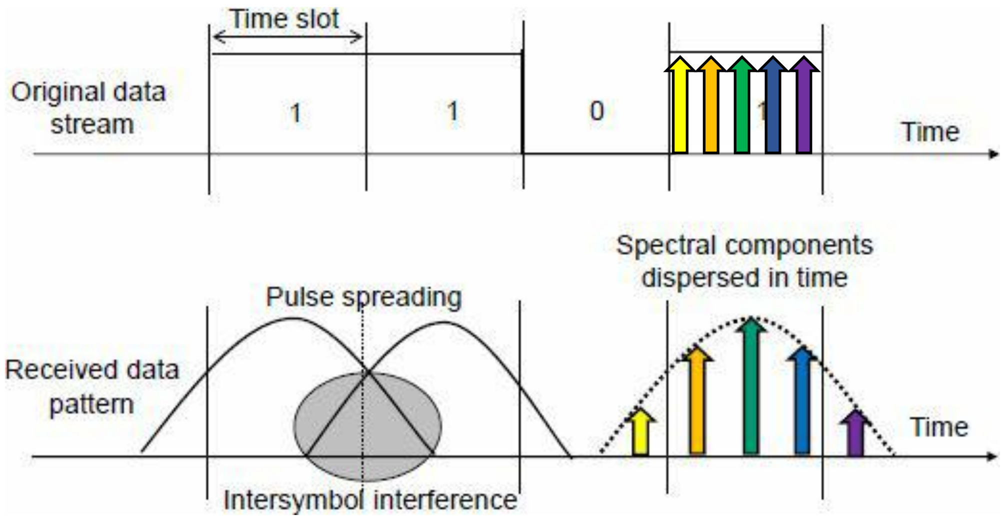
- In coherent optical systems, **intradyne reception** is employed: The Tx and LO lasers are **not** phase locked with each other (c.f. with **homodyne** reception, where the Tx and LO oscillators are locked to each other, usually with a Phase-Locked Loop circuit).
- Thus, they can have slightly different wavelengths, and due to the laser linewidth, uncorrelated random phase noise.
- The resulting baseband constellations after the coherent receiver exhibit:
  - A **constant** (or slowly varying) **rotation**, proportional to the **frequency offset** of the two lasers.
  - Rapidly varying, small rotations due to the **combined laser linewidths**.



# Optical Fiber Transmission Impairments

## Chromatic (Intramodal) Dispersion

- The refractive index, ***n(ω)***, is frequency dependent. Since the group velocity is $v_g = c/n(\omega)$, it depends on the refractive index, and therefore is also frequency-dependent.
- Lasers are *not* ideal monochromatic sources. Each information-carrying pulse contains a number of spectral components that travel at different group velocities through the fiber.
- The amount of the dispersion is proportional to the spectral width of the optical source.




- The digital pulses provide envelopes for the spectral content of the laser source.
- For a Tx containing a range of wavelengths $\Delta\lambda$, each spectral component propagates with different velocity through fiber length $L$, arriving at different times at the Rx and causing pulse broadening of $\Delta t_g$ ps:

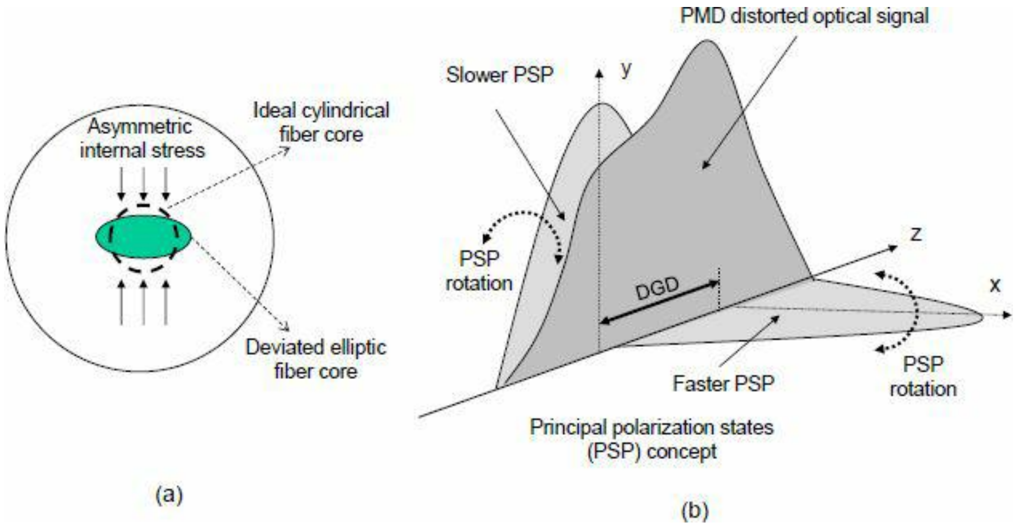
$$\Delta t_g = D \cdot L \cdot \Delta\lambda$$

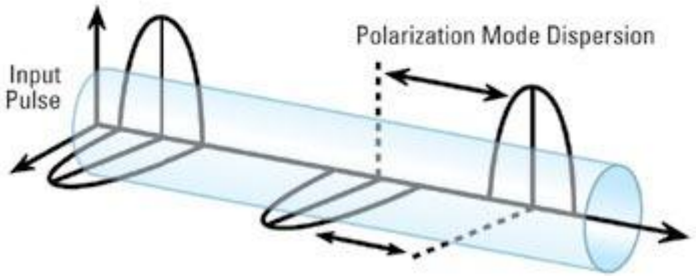
- D is the chromatic dispersion coefficient (ps/nm-km)

# Optical Fiber Transmission Impairments

## Polarization Mode Dispersion (PMD)

- Polarization mode dispersion (PMD) appears due to variations in the material and the shape of the core along the fiber length. Deviations give rise to the birefringence effect and the creation of two distinct orthogonal polarization modes (principal states of polarization, PSP), causing single-mode fibers to effectively become bimodal.
- The two polarizations see different refractive indices, and therefore travel at different speeds, causing a differential group delay, **DGD** between the slow and fast axes.



# Optical Fiber Transmission Impairments

## Polarization Mode Dispersion (PMD)

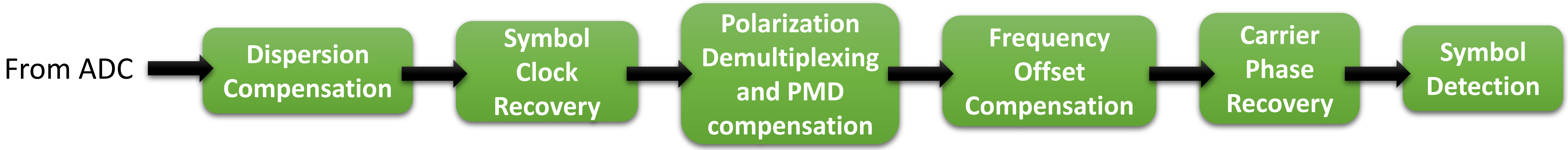
- The two polarization modes exchange the energy through the mode-coupling process during fiber propagation, caused by the external perturbation due to bending, twisting, lateral stress, and temperature variations.
- The degree of birefringence ($B = |n_x - n_y|$) changes randomly along the fiber, and we have a **stochastic rotation of the polarization states** and a stochastic distribution of energy among them.
- Thus PMD causes **stochastic pulse spreading**.
- Light polarization is analyzed using complex representation and the Jones matrix, which connects inputs and outputs carried over principal polarization states:

Delay acquired in the path

Power splitting of the signal between the 2 PSPs

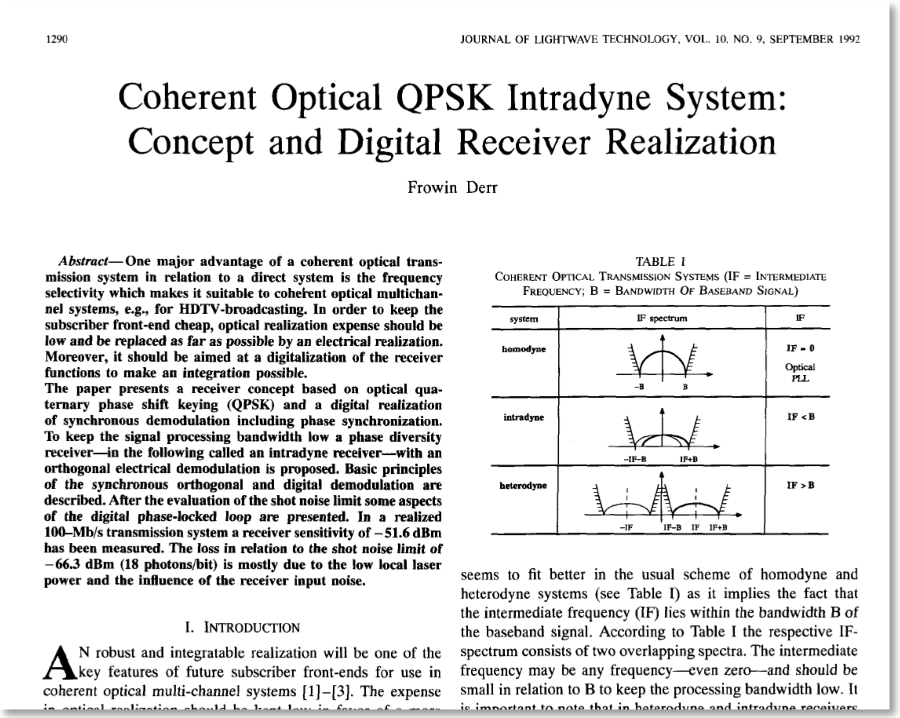
$$\begin{bmatrix} E_{x,out} \\ E_{y,out} \end{bmatrix} = [J] \cdot \begin{bmatrix} E_{x,in} \\ E_{y,in} \end{bmatrix} = \begin{bmatrix} \sqrt{\zeta} e^{j\Delta\tau/2} & -\sqrt{1-\zeta} \\ \sqrt{1-\zeta} & \sqrt{\zeta} e^{-j\Delta\tau/2} \end{bmatrix} \begin{bmatrix} E_{x,in} \\ E_{y,in} \end{bmatrix}$$

# Digital Signal Processing




## Why Use DSP?

- We use lasers as oscillators for the Tx and the Local Oscillator in the Rx, with intradyne detection (Tx and Rx are not phase locked).
- Unlike typical RF oscillators, it is not easy to perfectly lock two lasers in frequency and phase (e.g. with an electro-optical PLL), which is needed to demodulate the signal.
- Digital implementation of the phase locking is possible (e.g. a digital PLL or other techniques), but the baudrates (10-32 GBaud) at which optical systems work require huge processing power in the digital electronics.
- Thus, even though coherent intradyne detection is an old concept (1992 paper by Derr), it has been impractical until now.



1290 JOURNAL OF LIGHTWAVE TECHNOLOGY, VOL. 10, NO. 9, SEPTEMBER 1992

Coherent Optical QPSK Intradyne System: Concept and Digital Receiver Realization

Frowin Derr

*Abstract*—One major advantage of a coherent optical transmission system in relation to a direct system is the frequency selectivity which makes it suitable to coherent optical multichannel systems, e.g., for HDTV-broadcasting. In order to keep the subscriber front-end cheap, optical realization expense should be low and be replaced as far as possible by an electrical realization. Moreover, it should be aimed at a digitalization of the receiver functions to make an integration possible.
The paper presents a receiver concept based on optical quaternary phase shift keying (QPSK) and a digital realization of synchronous demodulation including phase synchronization. To keep the signal processing bandwidth low a phase diversity receiver—in the following called an intradyne receiver—with an orthogonal electrical demodulation is proposed. Basic principles of the synchronous orthogonal and digital demodulation are described. After the evaluation of the shot noise limit some aspects of the digital phase-locked loop are presented. In a realized 100–Mb/s transmission system a receiver sensitivity of −51.6 dBm has been measured. The loss in relation to the shot noise limit of −66.3 dBm (18 photons/bit) is mostly due to the low local laser power and the influence of the receiver input noise.

I. INTRODUCTION

An robust and integratable realization will be one of the key features of future subscriber front-ends for use in coherent optical multi-channel systems [1]–[3]. The expense

TABLE I
COHERENT OPTICAL TRANSMISSION SYSTEMS (IF = INTERMEDIATE FREQUENCY; B = BANDWIDTH OF BASEBAND SIGNAL)

| system | IF spectrum | IF |
|---|---|---|
| homodyne | −B B | IF = 0<br>Optical PLL |
| intradyne | −IF−B IF+B | IF < B |
| heterodyne | −IF IF−B IF IF+B | IF > B |

seems to fit better in the usual scheme of homodyne and heterodyne systems (see Table I) as it implies the fact that the intermediate frequency (IF) lies within the bandwidth B of the baseband signal. According to Table I the respective IF-spectrum consists of two overlapping spectra. The intermediate frequency may be any frequency—even zero—and should be small in relation to B to keep the processing bandwidth low. It

# Digital Signal Processing

From ADC → Dispersion Compensation → Symbol Clock Recovery → Polarization Demultiplexing and PMD compensation → Frequency Offset Compensation → Carrier Phase Recovery → Symbol Detection

## Why Use DSP?

- Recent developments in high-speed ADCs and ASICs have enabled the use of real-time DSP algorithms to demodulate Gbit/s coherent optical signals.
- By DSP processing it is possible to compensate for the “incoherence” (frequency offset and phase noise) of the Tx and LO lasers.
- Advanced RF/wireless comms concepts finally applicable for ultra-high speed optical comms.
- State-of-the-art: 100 Gbits/s optical transceivers are a commercial reality and deployed by major telecom operators today (DP-QPSK at 25 Gbaud symbol rate).

# Digital Signal Processing

From ADC → Dispersion Compensation → Symbol Clock Recovery → Polarization Demultiplexing and PMD compensation → Frequency Offset Compensation → Carrier Phase Recovery → Symbol Detection

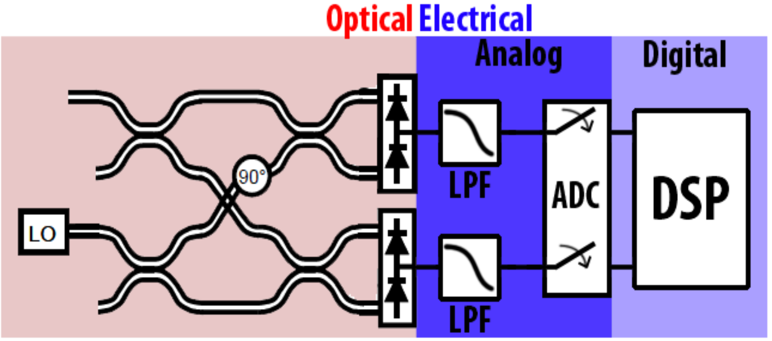
## Why Use DSP?

- DSP can also be used to compensate for  link impairments:
  - Chromatic Dispersion (CD)
  - Fiber nonlinearities
  - Bandwidth limitations
- Higher rates and longer transmission distances
- Universal transceivers using the same hardware in all parts of the network; rate and format is determined by the software.
- More transparent and upgradable networks
- Lower cost

# Digital Signal Processing

From ADC → Dispersion Compensation → Symbol Clock Recovery → Polarization Demultiplexing and PMD compensation → Frequency Offset Compensation → Carrier Phase Recovery → Symbol Detection

- The received signals from the photodiodes of the coherent receiver need to first be sampled.
- According to Nyquist, we need to sample at *at least* twice the symbol rate of the signal. E.g. for 28 GBaud we need at least 56 GSamples/s (**2 samples/symbol**).
- **N.B.:** The sampling rate (Rx) clock is **asynchronous** to the Tx clock. We are *not* sampling at the optimum pulse height, and the two clocks are *not* phase locked.



# Digital Signal Processing

From ADC → **Dispersion Compensation** → **Symbol Clock Recovery** → **Polarization Demultiplexing and PMD compensation** → **Frequency Offset Compensation** → **Carrier Phase Recovery** → **Symbol Detection**

- Chromatic Dispersion is a linear phenomenon, and deterministic (a function of the fiber characteristics, and length).

$$\Delta t_g = D \cdot L \cdot \Delta\lambda$$

- The fiber transfer function (considering only C.D.) is an all-pass filter (magnitude unchanged, while the phase is a function of $\boldsymbol{\omega}$).

Angular frequency ($2\pi f$)

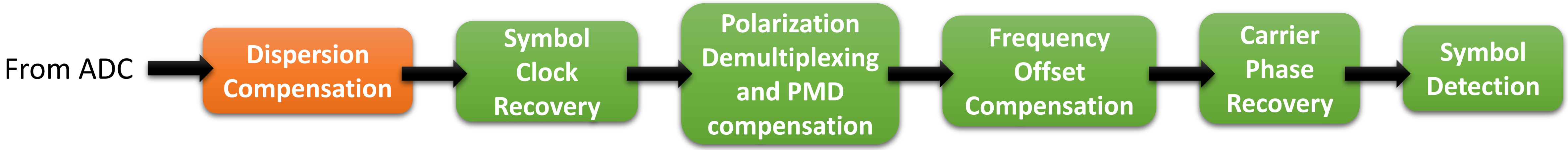
$$G(l,\omega) = \exp\left(-jD\frac{\lambda^2}{2\pi c}\frac{\omega^2}{2}l\right)$$

Fiber length

Fiber dispersion coefficient

Wavelength

# Digital Signal Processing

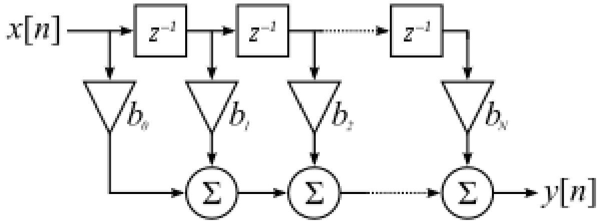



- Taking the inverse Fourier Transform of the frequency domain transfer function yields the time-domain transfer function.

$$G(l,\omega) = \exp(-jD\frac{\lambda^2}{2\pi c}\frac{\omega^2}{2}l)$$

$$g(z,t) = \sqrt{\frac{c}{jD\lambda^2 z}}\exp\left(j\frac{\pi c}{D\lambda^2 z}t^2\right)$$

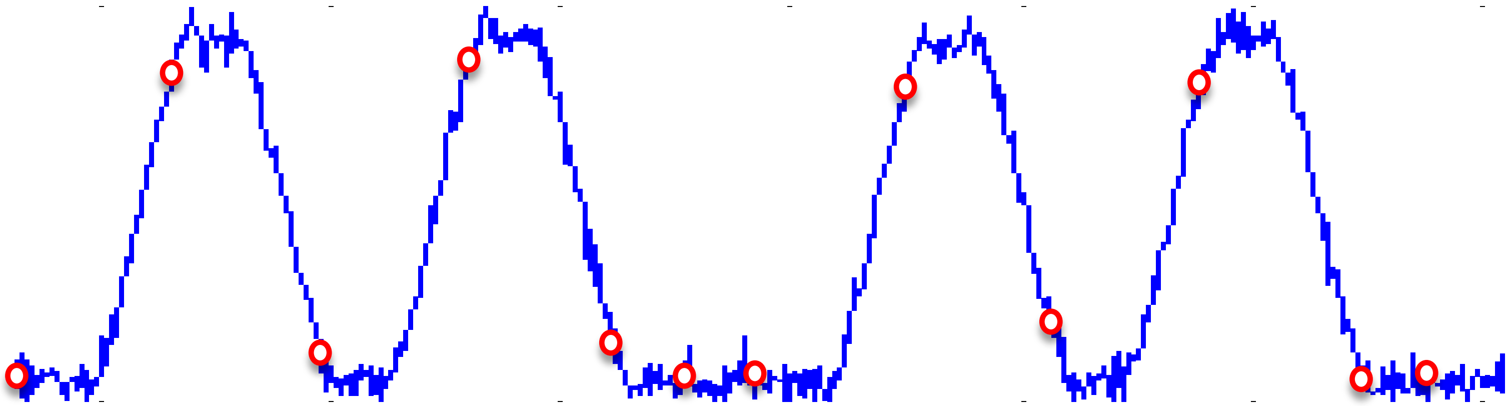
- This can then be easily approximated by a complex-coefficient **FIR filter** operating with **2 samples/symbol** that reverses the effect of chromatic dispersion (needs a negative sign in the exp() expression above).



# Digital Signal Processing

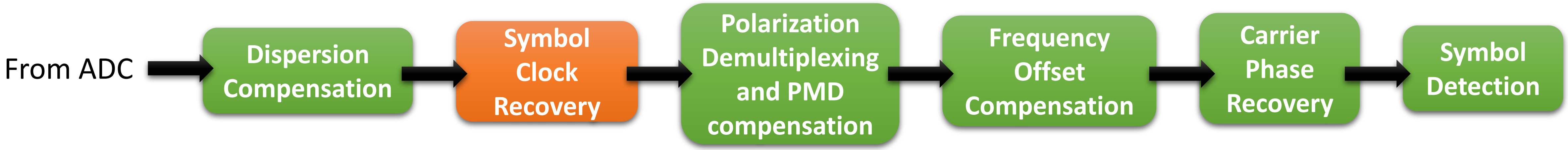
From ADC → Dispersion Compensation → Symbol Clock Recovery → Polarization Demultiplexing and PMD compensation → Frequency Offset Compensation → Carrier Phase Recovery → Symbol Detection

- After CD compensation (i.e. pulse dispersion is reversed), it is possible to acquire the clock of the data stream.
- The Tx and Rx symbol clocks are not synchronized. Since the ADC clock starts sampling at an arbitrary point in time, the samples it will obtain do not coincide with the optimum sampling point on the received waveform.
- The symbol clock recovery algorithm **finds the symbol clock frequency**, selects the **optimum sampling point** and resamples the data accordingly (normally outputs 2 samples/symbol).



Illustrating mis-timed ADC sampling (1 sample/symbol)

# Digital Signal Processing

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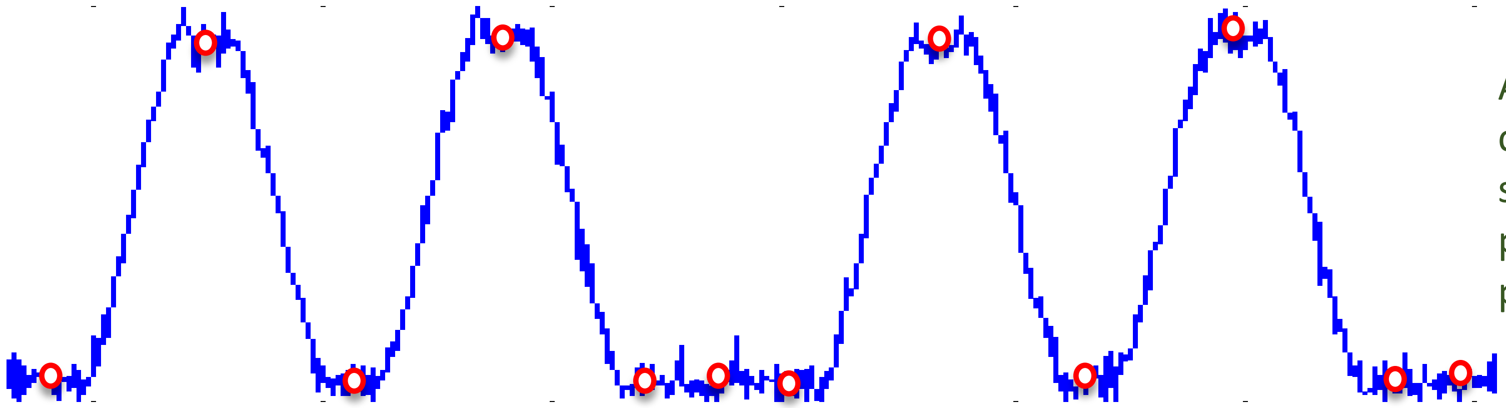


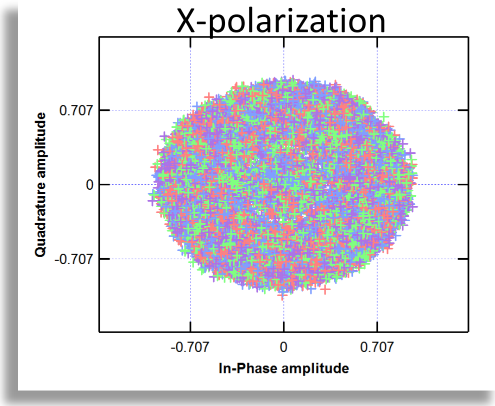
After sampling time correction, we obtain samples at the optimum points (the middle of the pulses)

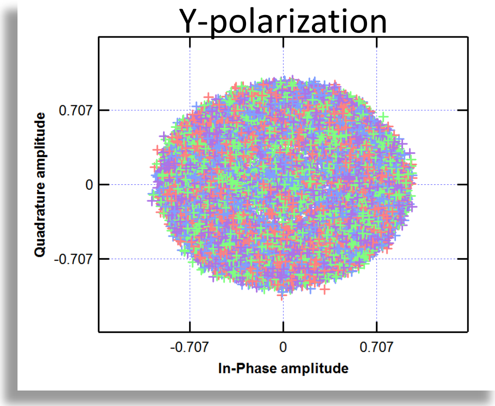
# Digital Signal Processing

From ADC → Dispersion Compensation → Symbol Clock Recovery → **Polarization Demultiplexing and PMD compensation** → Frequency Offset Compensation → Carrier Phase Recovery → Symbol Detection

- After acquiring the right symbol clock for each polarization, we have two signals (1 on each polarization), but with mixed data due to the rotated state of polarization (see PMD slide).

$$\begin{bmatrix} E_{x,out} \\ E_{y,out} \end{bmatrix} = [J] \cdot \begin{bmatrix} E_{x,in} \\ E_{y,in} \end{bmatrix} = \begin{bmatrix} \sqrt{\zeta}\, e^{j\Delta\tau/2} & -\sqrt{1-\zeta} \\ \sqrt{1-\zeta} & \sqrt{\zeta}\, e^{-j\Delta\tau/2} \end{bmatrix} \begin{bmatrix} E_{x,in} \\ E_{y,in} \end{bmatrix}$$



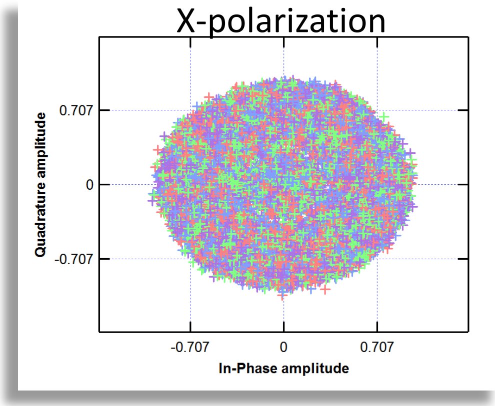



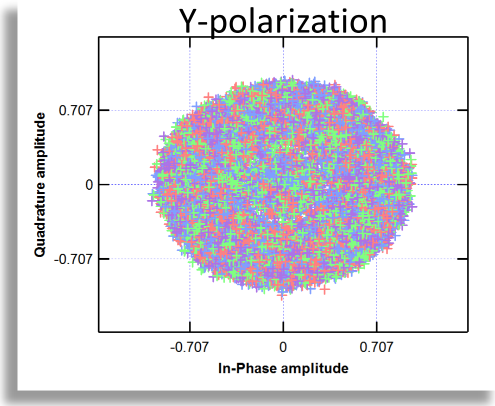
# Digital Signal Processing

From ADC → Dispersion Compensation → Symbol Clock Recovery → **Polarization Demultiplexing and PMD compensation** → Frequency Offset Compensation → Carrier Phase Recovery → Symbol Detection

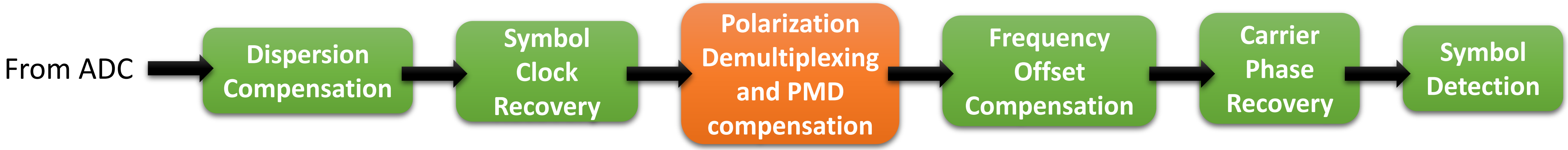
- We need to **estimate** and **reverse** the effect of the Jones matrix.

$$\begin{bmatrix} E_{x,out} \\ E_{y,out} \end{bmatrix} = [J] \cdot \begin{bmatrix} E_{x,in} \\ E_{y,in} \end{bmatrix} = \boxed{\begin{bmatrix} \sqrt{\zeta} e^{j\Delta\tau/2} & -\sqrt{1-\zeta} \\ \sqrt{1-\zeta} & \sqrt{\zeta} e^{-j\Delta\tau/2} \end{bmatrix}} \begin{bmatrix} E_{x,in} \\ E_{y,in} \end{bmatrix}$$



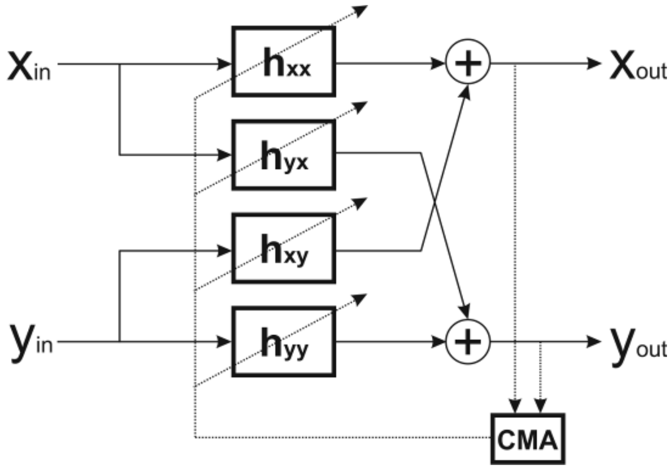



# Digital Signal Processing

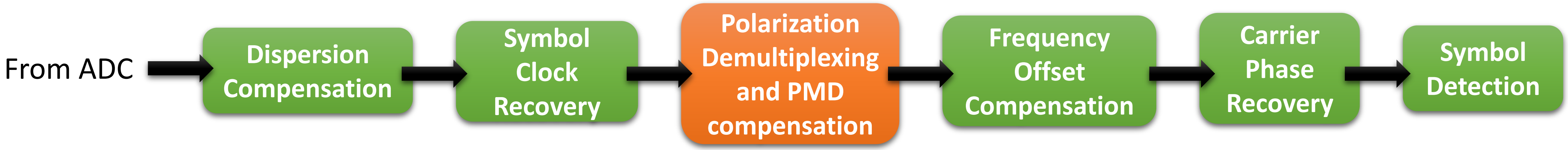



- We achieve this using a so-called FIR butterfly structure.
- **Adaptive equalization** is used (the **Constant Modulus Algorithm** is common) to find the optimum FIR taps that will find invert the Jones matrix for our channel.
- Because PMD is stochastic, the FIR filter taps are constantly being updated with every received symbol, to mitigate the effects of dispersion.

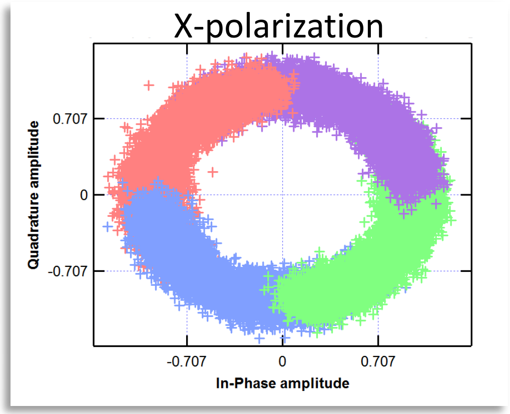
$$\begin{bmatrix} E_{x,out} \\ E_{y,out} \end{bmatrix} = [J] \cdot \begin{bmatrix} E_{x,in} \\ E_{y,in} \end{bmatrix} = \boxed{\begin{bmatrix} \sqrt{\zeta} e^{j\Delta\tau/2} & -\sqrt{1-\zeta} \\ \sqrt{1-\zeta} & \sqrt{\zeta} e^{-j\Delta\tau/2} \end{bmatrix}} \begin{bmatrix} E_{x,in} \\ E_{y,in} \end{bmatrix}$$

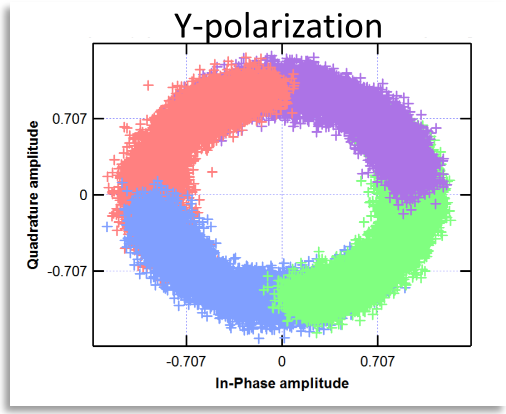


# Digital Signal Processing




- The result: Two signals which are separated (but are rotating due to carrier frequency offset and phase noise!).

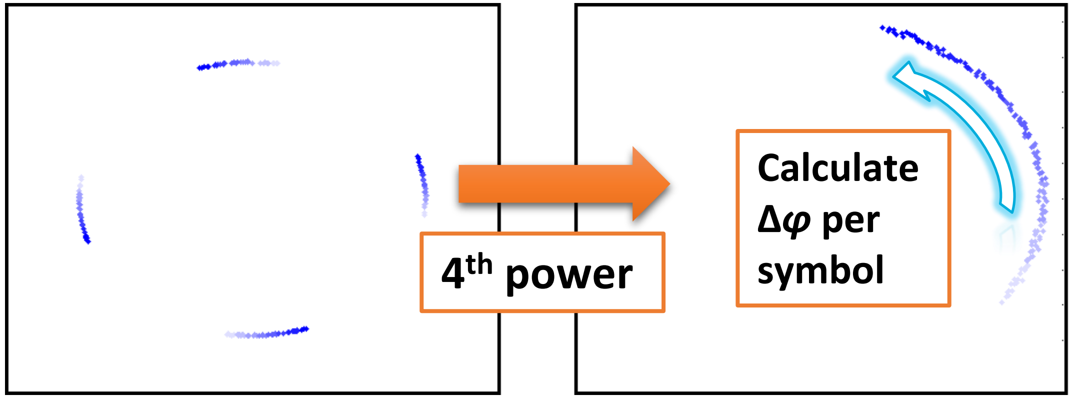


# Digital Signal Processing

From ADC → Dispersion Compensation → Symbol Clock Recovery → Polarization Demultiplexing and PMD compensation → Frequency Offset Compensation → Carrier Phase Recovery → Symbol Detection

- **Frequency offset estimation** and **carrier phase recovery** rely on the same principle: The idea is to estimate the rate of rotation of the constellation diagram, and then remove it.
- For frequency offset, the rotation we are trying to estimate is (more or less) constant.
- For phase noise, it is more rapid and needs to be tracked on shorter time scales.

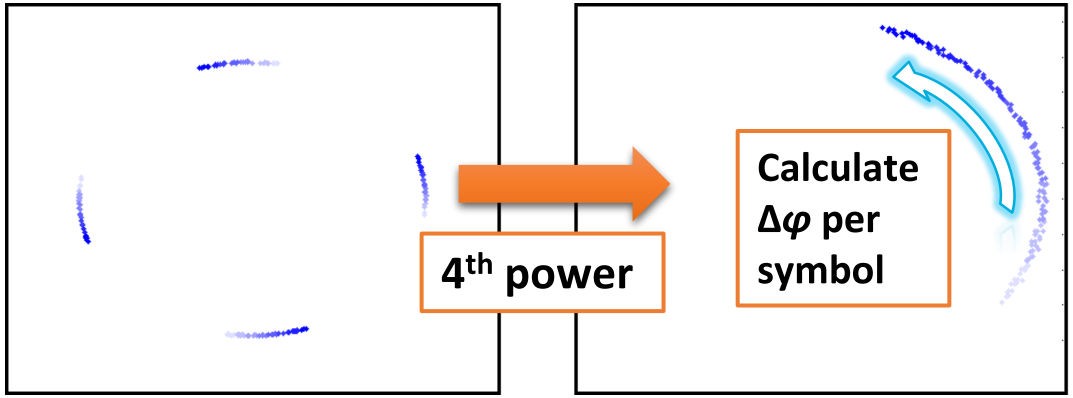



$$F_{offset} = \Delta\varphi/(2\pi T_s)$$

# Digital Signal Processing

From ADC → Dispersion Compensation → Symbol Clock Recovery → Polarization Demultiplexing and PMD compensation → Frequency Offset Compensation → Carrier Phase Recovery → Symbol Detection

- For QPSK, the most common approach for carrier recovery is the **Viterbi-Viterbi 4th power algorithm**.
- It essentially removes the modulation from the signal, leaving us with the carrier alone (a single, rotating point in the IQ plane).
- It is then straightforward to remove this rotation.

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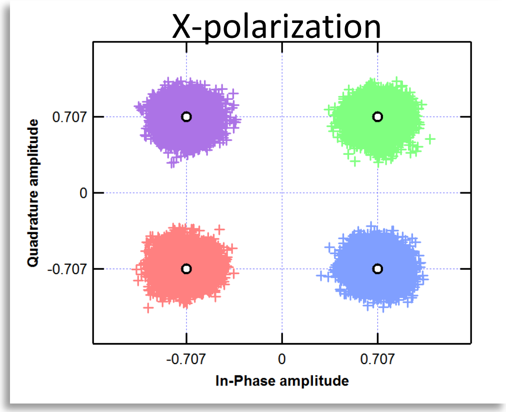
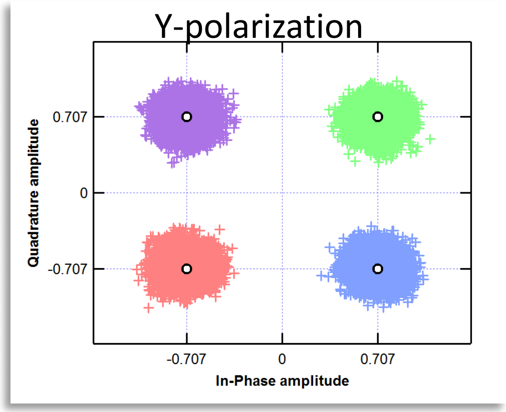
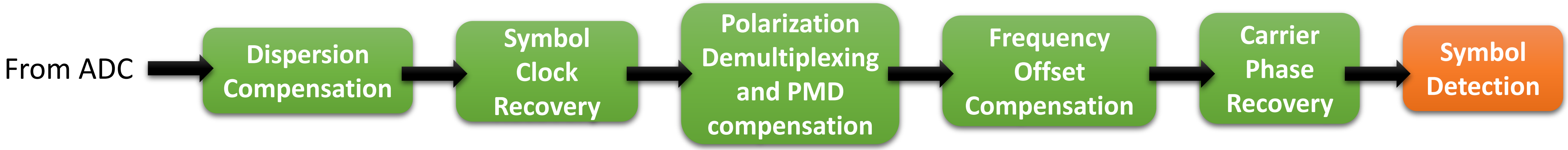
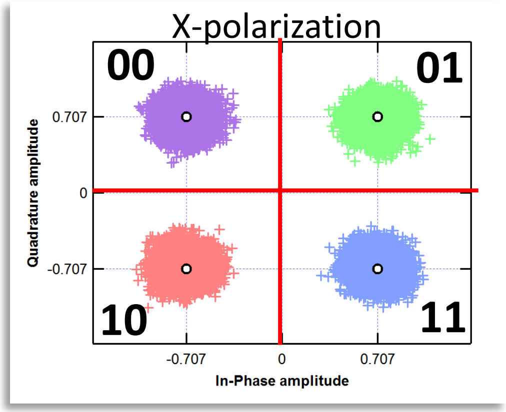


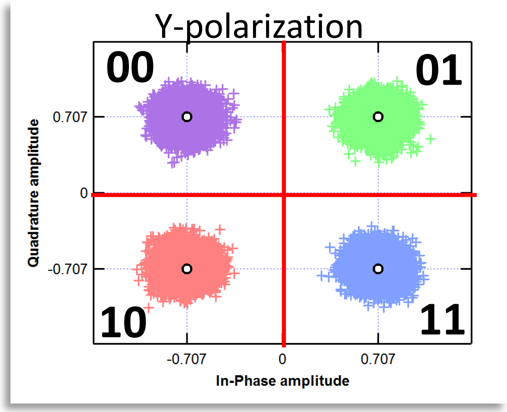



# Digital Signal Processing




- Finally, we can slice the received constellations using appropriate thresholds, and detect the symbols (and bits) transmitted through the fiber.



# Digital Signal Processing

From ADC → Dispersion Compensation → Symbol Clock Recovery → Polarization Demultiplexing and PMD compensation → Frequency Offset Compensation → Carrier Phase Recovery → Symbol Detection