What Casimir Energy can suggest about Space Time Foam?

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In the context of a model of space-time foam, made by N wormholes we discuss the possibility of having a foam formed by different configurations. An equivalence between Schwarzschild and Schwarzschild-Anti-de Sitter wormholes in terms of Casimir energy is shown. An argument to discriminate which configuration could represent a foamy vacuum coming from Schwarzschild black hole transition frequencies is used. The case of a positive cosmological constant is also discussed. Finally, a discussion involving charged wormholes leads to the conclusion that they cannot be used to represent a ground state of the foamy type.

Problem: How calculate the zero point energy (Z.P.E.) in pure gravity and what is the Casimir effect in such a configuration? The formulation of the Casimir effect in general is synthetized by the Eq.

$$E_{Casimir}\left[\partial\mathcal{M}\right] = E_0\left[\partial\mathcal{M}\right] - E_0\left[0\right], \tag{1}$$

where E_0 is the Z.P.E. and $\partial\mathcal{M}$ is a boundary. It is immediate to see that the Casimir energy involves a vacuum subtraction procedure and since this one is related to Z.P.E., we can extract information on the ground state. Space-time foam is a possible candidate for a ground state of quantum gravity. It was J. A. Wheeler who first conjectured that spacetime could be subjected to topology fluctuation at the Planck scale [1]. This means that spacetime undergoes a deep and rapid transformation in its structure. This changing spacetime is best known as space-time foam. However, in which way it is possible to construct such a fluctuating structure. One possibility comes by the computation of the following quantity

$$E\left(wormhole\right) = E\left(no-wormhole\right)$$

$$+\Delta E^{wormhole}_{no-wormhole|classical} + \Delta E^{wormhole}_{no-wormhole|1-loop}, \tag{2}$$

representing the total energy computed to one-loop in a wormhole background. $E\left(no-wormhole\right)$ is the reference space energy which, in the case of the Schwarzschild and RN wormhole, is flat space. $\Delta E^{wormhole}_{no-wormhole|classical}$ is the classical energy difference between the wormhole and no-wormhole configuration stored in the boundaries and finally $\Delta E^{wormhole}_{no-wormhole|1-loop}$ is the quantum correction to the classical term. One reason leading to Eq.(2) is in vacuum Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0. \tag{3}$$

The only spherically symmetric metrics solutions of Eq.(3) are the flat and Schwarzschild metric, respectively. The Schwarzschild metric can be thought as a wormhole with topology $S^2 \times R^1$ which asymptotically tend to the flat metric. Therefore, in this context, it is natural the comparison between the Schwarzschild wormhole and the flat space Z.P.E.. The inclusion of a negative and positive cosmological constant is straightforward. Thus the main reason leading to Eq.(2) is that the wormhole is a measure of the strong curvature of the gravitational field and the related

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Casimir energy is a measure of the vacuum energy. In a series of papers, we have used such an idea to concretely realize a model of space-time foam composed by a collection of N_w Schwarzschild wormholes [2–7]. A consequence of this model is that if we compute the area of the Schwarzschild black hole event horizon on a "*foam*" state $|\Psi_F\rangle$ we get

$$M = \frac{\sqrt{N}}{2l_p}\sqrt{\frac{\ln 2}{\pi}}, \tag{4}$$

namely the Schwarzschild black hole mass M has been *quantized* in terms of l_p [8–14]. This implies also that the level spacing of the transition frequencies is

$$\omega_0 = \Delta M = \left(8\pi M l_p^2\right)^{-1}\ln 2. \tag{5}$$

Note that for the Schwarzschild wormholes, Eq.(2) gives at its minimum

$$\Delta E_{N_w} = -N_w \frac{V}{64\pi^2}\frac{\Lambda^4}{e}, \tag{6}$$

obtained by computing the total Casimir energy

$$N_w \Delta E = N_w \left(E^{wormhole} - E^{No-wormhole}\right)$$

$$= \frac{N_w}{2\pi}\int_0^{+\infty} dp \int_0^{+\infty} dl \left(2l+1\right)\left[\left(\frac{d\delta_l^+\left(p\right)}{dp} + \frac{d\delta_l^-\left(p\right)}{dp}\right)\right] p. \tag{7}$$

The phase shift is defined as $\left(r \equiv r\left(x\right)\right)$

$$\delta_l^{\pm}\left(p\right) = \lim_{R\to+\infty}\left[\int_{r_h}^{x(R)} dx\sqrt{p^2 - \frac{l\left(l+1\right)}{r^2} - \tilde{V}^{\mp}\left(r\right)} - \int_{r_h}^{x(R)} dx\sqrt{p^2 - \frac{l\left(l+1\right)}{r^2}}\right], \tag{8}$$

where x is the proper distance from the throat and $\tilde{V}^{\mp}\left(r\right)$ is the curvature potential due to the wormhole. In Eq.(6) a cut-off Λ has been introduced to keep under control the U.V. divergence and V is a "local" volume defined by

$$V = 4\pi\int_{x(r_h)}^{x(r_0)} dx r^2. \tag{9}$$

r_h is the throat location (horizon) and $r_0 > r_h$ with $r_0 \neq \alpha r_h$ and α is a constant. It is interesting to see that Eq.(6) appears even for the Anti-de Sitter (AdS) case, described by the Schwarzschild-Anti-de Sitter (S-AdS) wormholes [15]. This means that our model of foam can be represented either by Schwarzschild wormholes or by S-AdS wormholes. If we compute the Schwarzschild black hole level spacing of transition frequencies in terms of S-AdS wormholes, we obtain

$$\omega_0 = \Delta M_S^{AdS} = \frac{9}{16}\left(8\pi M l_p^2\right)^{-1}\ln 2 \tag{10}$$

which is smaller with respect to ω_0 of Eq.(5). Thus we can use the difference in the spectrum to select the correct configuration of the foam constituents. Of course, nothing prevents to consider even a positive cosmological constant. The associated wormhole metric is described by the Schwarzschild-de Sitter metric which contains two throats: the wormhole throat and the cosmological throat. This last one contributes to the Z.P.E. leading to a value of [16]

$$\Delta E_{N_w} = -N_w \frac{V}{32\pi^2}\frac{\Lambda^4}{e}. \tag{11}$$

Nevertheless the case of the positive cosmological constant cannot be directly compared with the configuration spaces leading to Eq.(6), because the effect of quantum fluctuation is that of inducing a cosmological constant, which in the case of the Schwarzschild-de Sitter wormhole is included from the beginning. The situation seems more closely related to a sequence decay mechanism of the type

$$Flat\ Space$$

$$\Downarrow$$

$$Schwarzschild\ Spacetime\ Foam$$

$$\Downarrow$$

$$Schwarzschild - de\ Sitter\ Spacetime\ Foam.$$

Finally we wish to mention the case with a charge without a cosmological constant. In this situation we have the electric field contribution (magnetic field, respectively) to the Z.P.E.. Due to the presence of this non-gravitational field to Z.P.E., one finds that the Z.P.E. for charged wormholes is always higher than the neutral ones [17]. A similar computation in presence of a cosmological constant has not been done. However, the effect of the cosmological constant on Z.P.E. is to shift the vacuum position which is subsequently subtracted by the reference space. This is the main reason that leads to a common result for the AdS and the Schwarzschild wormholes. An important remark corroborating the foam vacuum is that a shift of the Z.P.E. with respect to the expected vacuum should not be there. The fact that one discovers a deviation from the expected result, even in absence of a renormalization process, is a clear signal of a bad vacuum choice.

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