CHAPTER FIVE

GAS TURBINES AND JET ENGINES

5.1 Introduction

History records over a century and a half of interest in and work on the gas turbine. However, the history of the gas turbine as a viable energy conversion device began with Frank Whittle's patent award on the jet engine in 1930 and his static test of a jet engine in 1937. Shortly thereafter, in 1939, Hans von Ohain led a German demonstration of jet-engine-powered flight, and the Brown Boveri company introduced a 4-MW gas-turbine-driven electrical power system in Neuchatel, Switzerland. The success of the gas turbine in replacing the reciprocating engine as a power plant for high-speed aircraft is well known. The development of the gas turbine was less rapid as a stationary power plant in competition with steam for the generation of electricity and with the spark-ignition and diesel engines in transportation and stationary applications. Nevertheless, applications of gas turbines are now growing at a rapid pace as research and development produces performance and reliability increases and economic benefits.

5.2 An Ideal Simple-Cycle Gas Turbine

The fundamental thermodynamic cycle on which gas turbine engines are based is called the *Brayton Cycle* or *Joule cycle*. A temperature-entropy diagram for this ideal cycle and its implementation as a *closed-cycle gas turbine* is shown in Figure 5.1. The cycle consists of an isentropic compression of the gas from state 1 to state 2; a constant pressure heat addition to state 3; an isentropic expansion to state 4, in which work is done; and an isobaric closure of the cycle back to state 1.

As Figure 5.1(a) shows, a compressor is connected to a turbine by a rotating shaft. The shaft transmits the power necessary to drive the compressor and delivers the balance to a power-utilizing load, such as an electrical generator. The turbine is similar in concept and in many features to the steam turbines discusssed earlier, except that it is designed to extract power from a flowing hot gas rather than from water vapor. It is important to recognize at the outset that the term "gas turbine" has a dual usage: It designates both the entire engine and the device that drives the compressor and the load. It should be clear from the context which meaning is intended. The equivalent term “combustion turbine” is also used occasionally, with the same ambiguity.

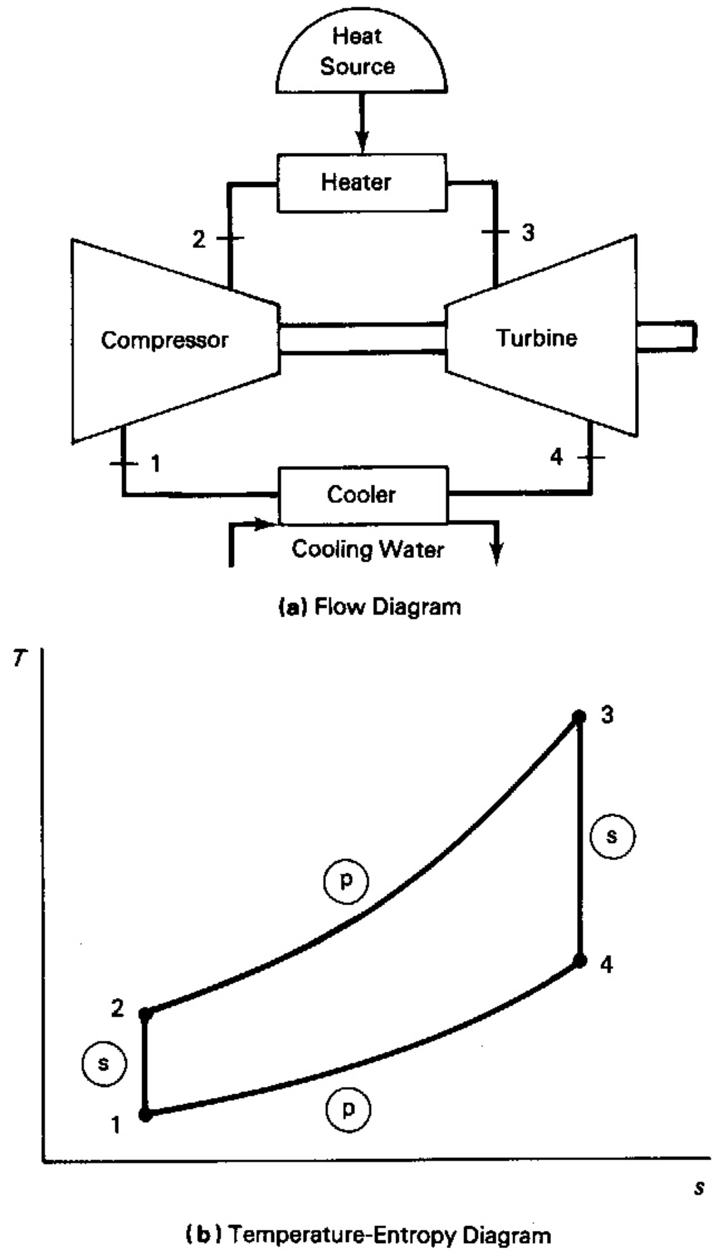

FIGURE 5.1 Closed-cycle gas turbine. (a) Flow diagram. (b) Temperature-entropy diagram.

Like the simple Rankine-cycle power plant, the gas turbine may be thought of as a device that operates between two pressure levels, as shown in Figure 5.1(b). The compressor raises the pressure and temperature of the incoming gas to the levels of p_2 and T_2. Expansion of the gas through the turbine back to the lower pressure at this point would be useless, because all the work produced in the expansion would be required to drive the compressor.

Instead, it is necessary to add heat and thus raise the temperature of the gas before expanding it in the turbine. This is achieved in the *heater* by heat transfer from an external source that raises the gas temperature to T_3, the turbine inlet temperature. Expansion of the hot gas through the turbine then delivers work in excess of that needed to drive the compressor. The turbine work exceeds the compressor requirement because the enthalpy differences, and hence the temperature differences, along isentropes connecting lines of constant pressure increase with increasing entropy (and temperature), as the figure suggests.

The difference between the turbine work, W_t, and the magnitude of the compressor work, $|W_c|$, is the net work of the cycle. The net work delivered at the output shaft may be used to drive an electric generator, to power a process compressor, turn an airplane propeller, or to provide mechanical power for some other useful activity.

In the closed-cycle gas turbine, the *heater* is a furnace in which combustion gases or a nuclear source transfer heat to the working fluid through thermally conducting tubes. It is sometimes useful to distinguish between internal and external combustion engines by whether combustion occurs in the working fluid or in an area separate from the working fluid, but in thermal contact with it. The combustion-heated, closed-cycle gas turbine is an example, like the steam power plant, of an *external combustion engine*. This is in contrast to *internal combustion engines*, such as automotive engines, in which combustion takes place in the working fluid confined between a cylinder and a piston, and in open-cycle gas turbines.

5.3 Analysis of the Ideal Cycle

The Air Standard cycle analysis is used here to review analytical techniques and to provide quantitative insights into the performance of an ideal-cycle engine. Air Standard cycle analysis treats the working fluid as a calorically perfect gas, that is, a perfect gas with constant specific heats evaluated at room temperature. In Air Standard cycle analysis the heat capacities used are those for air.

A gas turbine cycle is usually defined in terms of the compressor inlet pressure and temperature, p_1 and T_1, the *compressor pressure ratio*, $r = p_2/p_1$, and the turbine inlet temperature, T_3, where the subscripts correspond to states identified in Figure 5.1. Starting with the compressor, its exit pressure is determined as the product of p_1 and the compressor pressure ratio. The compressor exit temperature may then be determined by the familiar relation for an isentropic process in an ideal gas, Equation (1.19):

$$T_2 = T_1(p_2/p_1)^{(k-1)/k} \qquad [R \mid K] \qquad (5.1)$$

For the two isobaric processes, $p_2 = p_3$ and $p_4 = p_1$. Thus the *turbine pressure ratio*, p_3/p_4, is equal to the compressor pressure ratio, $r = p_2/p_1$. With the turbine inlet temperature T_3 known, the turbine discharge temperature can be determined from

$$T_4 = T_3/(p_2/p_1)^{(k-1)/k} \qquad [R \mid K] \qquad (5.2)$$

and the temperatures and pressures are then known at all the significant states.

Next, taking a control volume around the compressor, we determine the shaft work required by the compressor, w_c, assuming negligible heat losses, by applying the steady-flow energy equation:

$$0 = h_2 - h_1 + w_c$$

or

$$w_c = h_1 - h_2 = c_p(T_1 - T_2) \qquad [\text{Btu/lb}_m \mid \text{kJ/kg}] \qquad (5.3)$$

Similarly, for the turbine, the turbine work produced is

$$w_t = h_3 - h_4 = c_p (T_3 - T_4) \qquad [\text{Btu/lb}_m \mid \text{kJ/kg}] \qquad (5.4)$$

The net work is then

$$w_n = w_t + w_c = c_p (T_3 - T_4 + T_1 - T_2) \qquad [\text{Btu/lb}_m \mid \text{kJ/kg}] \qquad (5.5)$$

Now taking the control volume about the heater, we find that the heat addition per unit mass is

$$q_a = h_3 - h_2 = c_p (T_3 - T_2) \qquad [\text{Btu/lb}_m \mid \text{kJ/kg}] \qquad (5.6)$$

The cycle thermal efficiency is the ratio of the net work to the heat supplied to the heater:

$$\eta_{th} = w_n / q_a \qquad [\text{dl}] \qquad (5.7)$$

which by substitution of Equations (5.1), (5.2), (5.5), and (5.6) may be simplified to

$$\eta_{th} = 1 - (p_2/p_1)^{-(k-1)/k} \qquad [\text{dl}] \qquad (5.8)$$

It is evident from Equation (5.8) that increasing the compressor pressure ratio increases thermal efficiency.

Another parameter of great importance to the gas turbine is the *work ratio*, $w_t / |w_c|$. This parameter should be as large as possible, because a large amount of the power delivered by the turbine is required to drive the compressor, and because the engine net work depends on the excess of the turbine work over the compressor work. A little algebra will show that the work ratio $w_t/|w_c|$ can be written as:

$$w_t/|w_c| = (T_3 / T_1) / (p_2 / p_1)^{(k-1)/k} \qquad [\text{dl}] \qquad (5.9)$$

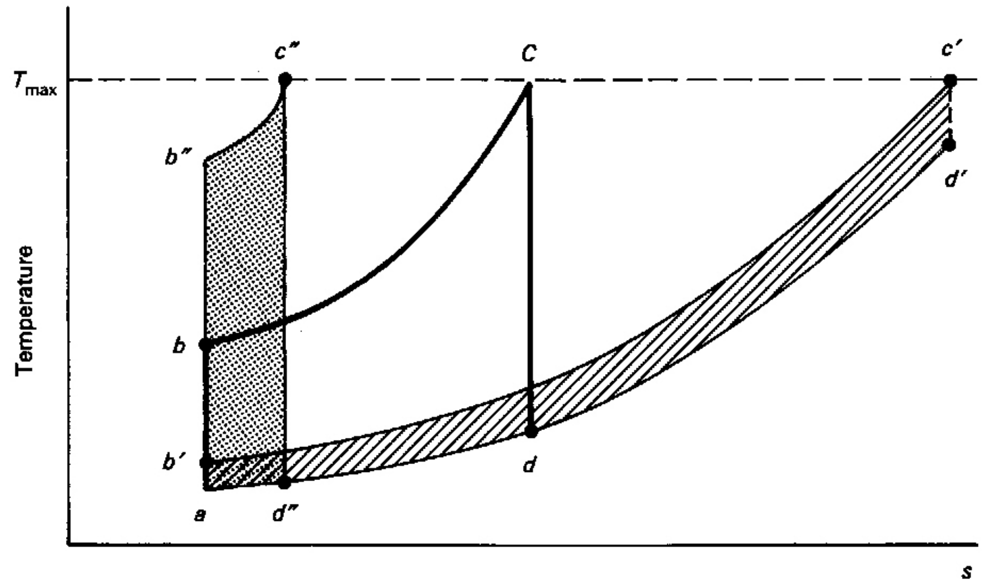

FIGURE 5.2 A family of Brayton cycles for fixed compressor and turbine inlet temperatures.

Note that, for the ideal cycle, the thermal efficiency and the work ratio depend on only two independent parameters, the compressor pressure ratio and the ratio of the turbine and compressor inlet temperatures. It will be seen that these two design parameters are of utmost importance for all gas turbine engines.

Equation (5.9) shows that the work ratio increases in direct proportion to the ratio T_3/T_1 and inversely with a power of the pressure ratio. On the other hand, Equation (5.8) shows that thermal efficiency increases with increased pressure ratio. Thus, the desirability of high turbine inlet temperature and the necessity of a tradeoff involving pressure ratio is clear. Equation (5.9) also suggests that increases in the ratio T_3/T_1 allow the compressor pressure ratio to be increased without reducing the work ratio. This is indicative of the historic trend by which advances in materials allow higher turbine inlet temperatures and therefore higher compressor pressure ratios.

It was shown in Chapter 1 that the area of a reversible cycle plotted on a T–s diagram gives the net work of the cycle. With this in mind, it is interesting to consider a family of cycles in which the compressor inlet state, a, and turbine inlet temperatures are fixed, as shown in Figure 5.2. As the compressor pressure ratio p_b/p_a approaches 1, the cycle area and hence the net work approach 0, as suggested by the shaded cycle labeled with single primes. At the other extreme, as the compressor pressure ratio approaches its maximum value, the net work also approaches 0, as in the cycle denoted by double primes. For intermediate pressure ratios, the net work is large and positive, indicating that there is a unique value of compressor pressure ratio that maximizes the net work. Such information is of great significance in gas turbine design,

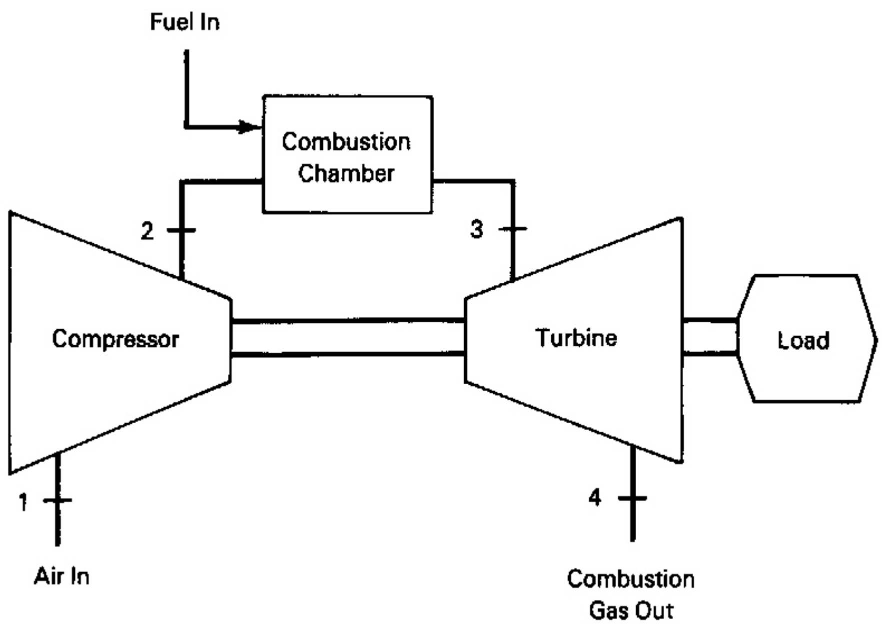

FIGURE 5.3 Open-cycle implementation of the Brayton cycle.

because it indicates the pressure ratio that yields the highest power output for given turbomachine inlet temperatures and mass flow rate. This is an important approach to the pressure ratio tradeoff mentioned earlier. It will be considered from an analytic viewpoint for a more realistic gas turbine model in a later section.

Up to this point the discussion has focused on the closed-cycle gas turbine, an external combustion or nuclear-heated machine that operates with a circulating fixed mass of working fluid in a true cyclic process. In fact, the same Air Standard cycle analysis may be applied to the *open-cycle gas turbine.* The open cycle operates with atmospheric air that is pressurized by the compressor and then flows into a combustion chamber, where it oxidizes a hydrocarbon fuel to produce a hot gas that drives the turbine. The turbine delivers work as in the closed cycle, but the exiting combustion gases pass into the atmosphere, as they must in all combustion processes.

A diagram of the cycle implementation is shown in Figure 5.3. Clearly, the open-cycle gas turbine is an *internal combustion engine,* like the automotive engine. Note that the diagram is consistent with Figure 5.1 and all the preceding equations in this chapter. This is true because (1) the atmosphere serves as an almost infinite source and sink that may be thought of as closing the cycle, and (2) the energy released by combustion has the same effect as the addition of external heat in raising the temperature of the gas to the turbine inlet temperature. A cutaway of an open-cycle utility gas turbine is presented in Figure 5.4.

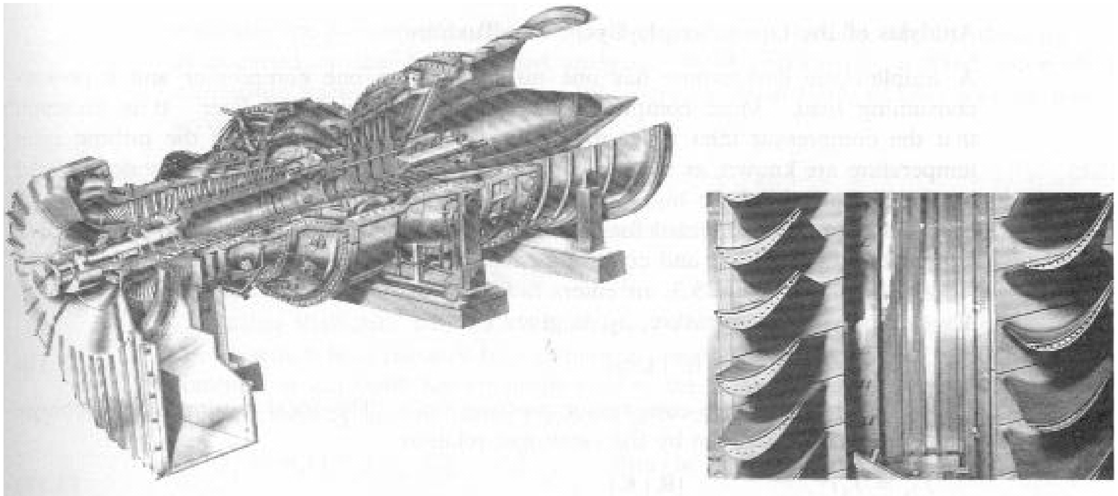

FIGURE 5.4 Cutaway of single-shaft combustion turbine. Air inlet and power takeoff flange are on the left, and combustion gas exit is at the right. The inset shows holes in the ends of the turbine rotor blades for exit of blade-cooling air. (Courtesy of Westinghouse Canada.)

5.4 Realistic Simple-Cycle Gas Turbine Analysis

The preceding analysis of the Air Standard cycle assumes perfect turbomachinery, an unachievable but meaningful ideal, and room-temperature heat capacities. Realistic quantitative performance information can be obtained by taking into account efficiencies of the compressor and the turbine, significant pressure losses, and more realistic thermal properties.

Properties for Gas Turbine Analysis

It is pointed out in reference 1 that accurate gas turbine analyses may be performed using constant heat capacities for both air and combustion gases. This appears to be a specialization of a method devised by Whittle (ref. 4). The following properties are therefore adopted for all gas turbine analyses in this book:

Air:

$c_p = 0.24$ Btu/lb$_m$-R or 1.004 kJ /kg-K

$k = 1.4$ implies $k/(k-1) = 3.5$

Combustion gas:

$c_{p,g} = 0.2744$ Btu/lb$_m$-R or 1.148 kJ /kg-K

$k_g = 1.333$ implies $k_g/(k_g - 1) = 4.0$

The properties labeled as combustion gas above are actually high-temperature-air properties. Because of the high air-fuel ratio required by gas turbines, the thermodynamic properties of gas turbine combustion gases usually differ little from those of high-temperature air. Thus the results given below apply equally well to closed-cycle machines using air as the working fluid and to open-cycle engines.

Analysis of the Open Simple-Cycle Gas Turbine

A simple-cycle gas turbine has one turbine driving one compressor and a power-consuming load. More complex configurations are discussed later. It is assumed that the compressor inlet state, the compressor pressure ratio, and the turbine inlet temperature are known, as before. The turbine inlet temperature is usually determined by the limitations of the high-temperature turbine blade material. Special metals or ceramics are usually selected for their ability to withstand both high stress at elevated temperature and erosion and corrosion caused by undesirable components of the fuel.

As shown in Figure 5.3, air enters the compressor at a state defined by T_1 and p_1. The compressor exit pressure, p_2, is given by

$$p_2 = rp_1 \qquad [\mathrm{lb_f/ft^2} \mid \mathrm{kPa}] \qquad (5.10)$$

where r is the compressor pressure ratio. The ideal compressor discharge temperature, T_{2s} is given by the isentropic relation

$$T_{2s} = T_1\, r^{(k-1)/k} \qquad [\mathrm{R} \mid \mathrm{K}] \qquad (5.11)$$

The *compressor isentropic efficiency*, defined as the ratio of the compressor isentropic work to the actual compressor work with both starting at the same initial state and ending at the same pressure level, may be written as

$$\eta_c = (h_1 - h_{2s})/(h_1 - h_2) = (T_1 - T_{2s})/(T_1 - T_2) \qquad [\mathrm{dl}] \qquad (5.12)$$

Here the steady-flow energy equation has been applied to obtain expressions for the work for an irreversible adiabatic compressor in the denominator and for an isentropic compressor in the numerator. Solving Equation (5.12) for T_2, we get as the actual compressor discharge temperature:

$$T_2 = T_1 + (T_{2s} - T_1)/\eta_c \qquad [\mathrm{R} \mid \mathrm{K}] \qquad (5.13)$$

Equation (5.3) then gives the work needed by the compressor, w_c:

$$w_c = c_p(T_1 - T_2) = c_p(T_1 - T_{2s})/\eta_c \qquad [\mathrm{Btu/lb_m} \mid \mathrm{kJ/kg}] \qquad (5.14)$$

Note that the compressor work is negative, as required by the sign convention that defines work as positive if it is produced by the control volume. The compressor power requirement is, of course, then given by $m_a w_c$ [Btu/hr | kW], where m_a is the compressor mass flow rate [lb_m / hr | kg / s].

After leaving the compressor at an elevated pressure and temperature, the air then enters the combustion chamber, where it completely oxidizes a liquid or a gaseous fuel injected under pressure. The combustion process raises the combustion gas temperature to the turbine inlet temperature T_3. One of the goals of combustion chamber design is to minimize the pressure loss from the compressor to the turbine. Ideally, then, $p_3 = p_2$, as assumed by the Air Standard analysis. More realistically, a fixed value of the combustor fractional pressure loss, fpl, (perhaps about 0.05 or 5%) may be used to account for burner losses:

$$\mathrm{fpl} = (p_2 - p_3)/p_2 \qquad [\mathrm{dl}] \qquad (5.15)$$

Then the turbine inlet pressure may be determined from

$$p_3 = (1 - \mathrm{fpl})\, p_2 \qquad [\mathrm{lb_f/ft^2} \mid \mathrm{kPa}] \qquad (5.16)$$

Rather than deal with its complexities, we may view the combustion process simply as one in which heat released by exothermic chemical reaction raises the temperature of combustion gas (with hot-air properties) to the turbine inlet temperature. The rate of heat released by the combustion process may then be expressed as:

$$Q_a = m_a(1 + f)c_{p,g}(T_3 - T_2) \qquad [\mathrm{Btu/hr} \mid \mathrm{kW}] \qquad (5.17)$$

where f is the mass fuel-air ratio. The term $m_a(1 + f)$ is seen to be the sum of the air and fuel mass flow rates, which also equals the mass flow rate of combustion gas. For gas turbines it will be seen later that f is usually much less than the stoichiometric fuel-air ratio and is often neglected with respect to 1 in preliminary analyses.

The turbine in the open-cycle engine operates between the pressure p_3 and atmospheric pressure, $p_4 = p_1$, with an inlet temperature of T_3. If the turbine were isentropic, the discharge temperature would be

$$T_{4s} = T_3(p_4/p_3)^{(kg-1)/kg} \qquad [\mathrm{R} \mid \mathrm{K}] \qquad (5.18)$$

From the steady-flow energy equation, the turbine work can be written as

$$w_t = c_{p,g}(T_3 - T_4) = \eta_t c_{p,g}(T_3 - T_{4s}) \qquad [\mathrm{Btu/lb_m} \mid \mathrm{kJ/kg}] \qquad (5.19)$$

referenced to unit mass of combustion gas, and where η_t is the turbine isentropic efficiency. The turbine power output is then $m_a(1 + f)w_t$, where, as seen earlier, $m_a(1 + f)$ is the mass flow rate of combustion gas flowing through the turbine. The net work based on the mass of air processed and the net power output of the gas turbine, P_n, are then given by

$$w_n = (1 + f)w_t + w_c \qquad [\text{Btu/lb}_m \text{ air}|\ \text{kJ/kg air}] \qquad (5.20)$$

and

$$P_n = m_a\,[(1 + f)w_t + w_c\,] \qquad [\text{Btu/hr}\ |\ \text{kW}] \qquad (5.21)$$

and the thermal efficiency of the engine is

$$\eta_{th} = P_n/Q_a \qquad [\text{dl}] \qquad (5.22)$$

EXAMPLE 5.1

A simple-cycle gas turbine has 86% and 89% compressor and turbine efficiencies, respectively, a compressor pressure ratio of 6, a 4% fractional pressure drop in the combustor, and a turbine inlet temperature of 1400°F. Ambient conditions are 60°F and one atmosphere. Determine the net work, thermal efficiency, and work ratio for the engine. Assume that the fuel-mass flow rate is negligible compared with the air flow rate.

Solution

The notation for the solution is that of Figure 5.3. The solution details are given in Table 5.1 in a step-by-step spreadsheet format. Each line presents the parameter name, symbol, and units of measure; its value; and the right-hand side of its specific determining equation.

TABLE 5.1 Spreadsheet Solution to Example 5.1

Example 5-1: Simple cycle gas turbine calculation

	VALUE	EQUATION
air isentropic exponent = k	1.40	given
(k-1)/k = e,a	0.2857	(k-1)/k
compressor inlet temperature = T1,R	520.00	given
compressor efficiency = etac	0.86	given
compressor pressure ratio = r	6.00	given
compressor isentropic exit temp. = T2s,R	867.63	T1r^e,a
compressor true exit temp. = T2,R	924.22	T1+(T2s-T1)/etac
compressor work = Wc,Btu/lb	-97.01	0.24(T1-T2)
combustor fractional pressure drop = fpl	0.04	given
turbine pressure ratio = rt	5.76	r(1-fpl)
turbine inlet temp = T3,R	1860.00	given
hot gas isentropic exponent = kg	1.33	given
(kg-1)/kg = e,g	0.25	(kg-1)/kg
hot gas heat capacity = cp,g, Btu/lb-R	0.2744	given
combustor heat addition = Qa, Btu/lb	256.78	cp,g(T3-T2)
turbine isentropic exit temp. = T4s,R	1200.63	T3/(rt)^e,g
turbine isentropic efficiency = etat	0.89	given
turbine true exit temp. = T4,R	1273.16	T3-etat(T3-T4s)
turbine work = Wt,Btu/lb	161.03	cp,g(T3-T4)
net work = Wn,Btu/lb	64.02	Wt+Wc
thermal efficiency	0.25	Wn/Qa
work ratio	1.66	Wt/|Wc|

When an entire cycle is to be analyzed, it is best to start at the compressor with the inlet conditions and proceed to calculate successive data in the clockwise direction on the T-s diagram. The compressor isentropic and actual discharge temperatures and work are determined first using Equations (5.11), (5.13), and (5.14). The turbine pressure ratio is determined next, accounting for the combustor pressure loss, using Equation (5.16). The isentropic relation, Equation (5.18), gives the isentropic turbine exit temperature, and the turbine efficiency and Equation (5.19) yields the true turbine exit temperature and work. Once all the turbomachine inlet and exit temperatures are known, other cycle parameters are easily determined, such as the combustor heat transfer, net work, thermal efficiency, and work ratio.

An important observation may be made on the basis of this analysis regarding the magnitude of the compressor work with respect to the turbine work. Much of the turbine work is required to drive the compressor. Compare the work ratio of 1.66, for example, with the much higher values for the steam cycles of Chapter 2 (the Rankine-cycle pumps have the same function there as the compressor here). Example 2.4 for the Rankine cycle with a 90% turbine efficiency has a work ratio of 77.2. Thus the gas turbine's pressurization handicap relative to the Rankine cycle is substantial.

The unimpressive value of the thermal efficiency of the example gas turbine, 25% (not typical of the current state of the art) compares with a Carnot efficiency for the same cycle temperature extremes of 72% The large amount of compressor work required clearly contributes to this weak performance. Nevertheless, current gas turbines are competitive with many other engines on an efficiency basis, and have advantages such as compactness and quick-start capability relative to Rankine cycle power plants. One approach to the improvement of thermal efficiency of the gas turbine will be addressed later in Section 5.5. First let's look at what can be done about gas turbine work.

Maximizing the Net Work of the Cycle

Using Equations (5.14) and (5.19), we can rewrite the cycle net work as

$$w_n = \eta_t c_{p,g} (T_3 - T_{4s}) - c_p(T_{2s} - T_1)/\eta_c \qquad \text{[Btu/lb}_m \text{ | kJ/kg]} \qquad (5.23)$$

where the fuel-air ratio has been neglected with respect to 1. In the following, the combustor pressure losses and the distinction between hot-gas and air heat capacities will be neglected but the very important turbomachine efficiencies are retained. Nondimensionalizing the net work with the constant c_pT_1 we get:

$$w_n/c_pT_1 = \eta_t (T_3/T_1)(c_{p,g}/c_p)(1 - r^{-(k-1)/k}) + (1 - r^{(k-1)/k})/\eta_c \qquad \text{[dl]} \qquad (5.24)$$

By differentiating w_n with respect to the compressor pressure ratio r and setting the

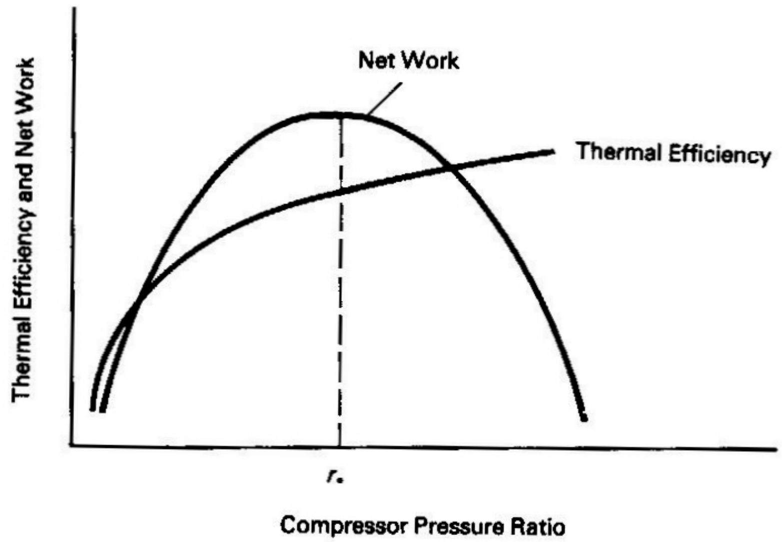

FIGURE 5.5 The influence of compressor pressure ratio on cycle efficiency and net work.

result equal to 0, we obtain an equation *for* r^*, the value of r that maximizes the net work with fixed turbomachine efficiencies and with a constant ratio of the temperatures of the turbomachine inlets, T_3/T_1. For constant gas properties throughout, the result is

$$r^* = (\eta_c\ \eta_t\ T_3/T_1)^{k/2(k-1)} \quad \text{[dl]} \qquad (5.25)$$

This relation gives a specific value for the compressor pressure ratio that defines an optimum cycle, in the sense of the discussion of Figure 5.2. There it was established qualitatively that a cycle with maximum net work exists for a given value of T_3/T_1. Equation (5.25) defines the condition for this maximum and generalizes it to include turbomachine inefficiency.

The pressure ratio r^* given by Equation (5.25) increases with increasing turbomachine efficiencies and with T_3/T_1. This is a clear indicator that increasing turbine inlet temperature favors designs with higher compressor pressure ratios. This information is important to the gas turbine designer but does not tell the whole design story. There are other important considerations; for example, (1) compressors and turbines become more expensive with increasing pressure ratios, and (2) the pressure ratio that maximizes thermal efficiency is different from that given by Equation (5.25). Figure 5.5 shows the influence of compressor pressure ratio on both efficiency and net work and the position of the value given by Equation (5.25). Thus, when all factors are taken into account, the final design pressure ratio is likely to be in the vicinity of, but not necessarily identical to, r^*.

5.5 Regenerative Gas Turbines

It was shown in Chapter 2 that the efficiency of the Rankine cycle could be improved by an internal transfer of heat that reduces the magnitude of external heat addition, a feature known as regeneration. It was also seen in Chapter 2 that this is accomplished conveniently in a steam power plant by using a heat exchanger known as a feedwater heater.

Examination of Example 5.1 shows that a similar opportunity exists for the gas turbine cycle. The results show that the combustion process heats the incoming air from 924°R to 1860°R and that the gas turbine exhausts to the atmosphere at 1273°R. Thus a maximum temperature potential of 1273 – 924 = 349°F exists for heat transfer. As in the Rankine cycle, this potential for regeneration can be exploited by incorporation of a heat exchanger. Figure 5.6 shows a gas turbine with a counterflow heat exchanger that extracts heat from the turbine exhaust gas to preheat the compressor discharge air to T_c ahead of the combustor. As a result, the temperature rise in the combustor is reduced to $T_3 - T_c$, a reduction reflected in a direct decrease in fuel consumed.

Note that the compressor and turbine inlet and exit states can be the same as for a simple cycle. In this case the compressor, turbine, and net work as well as the work ratio are unchanged by incorporating a heat exchanger.

The *effectiveness* of the heat exchanger, or *regenerator*, is a measure of how well it uses the available temperature potential to raise the temperature of the compressor discharge air. Specifically, it is the actual rate of heat transferred to the air divided by the maximum possible heat transfer rate that would exist if the heat exchanger had infinite heat transfer surface area. The actual heat transfer rate to the air is $mc_p(T_c - T_2)$, and the maximum possible rate is $mc_p(T_4 - T_2)$. Thus the *regenerator effectiveness* can be written as

$$\eta_{reg} = (T_c - T_2)/(T_4 - T_2) \quad \text{[dl]} \qquad (5.26)$$

and the combustor inlet temperature can be written as

$$T_c = T_2 + \eta_{reg}(T_4 - T_2) \quad \text{[R | K]} \qquad (5.27)$$

It is seen that the combustor inlet temperature varies from T_2 to T_4 as the regenerator effectiveness varies from 0 to 1. The regenerator effectiveness increases as its heat transfer area increases. Increased heat transfer area allows the cold fluid to absorb more heat from the hot fluid and therefore leave the exchanger with a higher T_c.

On the other hand, increased heat transfer area implies increased pressure losses on both air and gas sides of the heat exchanger, which in turn reduces the turbine pressure ratio and therefore the turbine work. Thus, increased regenerator effectiveness implies a tradeoff, not only with pressure losses but with increased heat exchanger size and complexity and, therefore, increased cost.

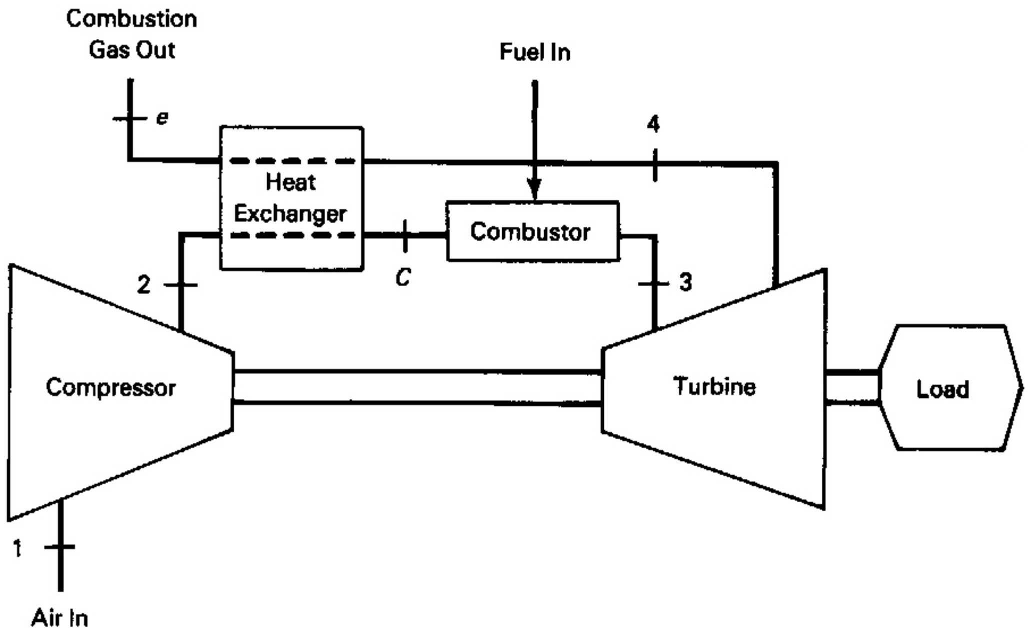

FIGURE 5.6 Regenerative gas turbine.

The exhaust gas temperature at the exit of the heat exchanger may be determined by applying the steady-flow energy equation to the regenerator. Assuming that the heat exchanger is adiabatic and that the mass flow of fuel is negligible compared with the air flow, and noting that no shaft work is involved, we may write the steady-flow energy equation for two inlets and two exits as

$$q = 0 = h_e + h_c - h_2 - h_4 + w = c_{p,g}T_e + c_pT_c - c_pT_2 - c_{p,g}T_4 + 0$$

Thus the regenerator combustion-gas-side exit temperature is:

$$T_e = T_4 - (c_p/c_{p,g})(T_c - T_2) \qquad [\text{R} \mid \text{K}] \qquad (5.28)$$

While the regenerator effectiveness does not appear explicitly in Equation (5.28), the engine exhaust temperature is reduced in proportion to the air temperature rise in the regenerator, which is in turn proportional to the effectiveness. The dependence of the exhaust temperature on η_{reg} may be seen directly by eliminating T_c from Equation (5.28), using Equation (5.27) to obtain

$$T_4 - T_e = \eta_{reg}\,(c_p/c_{p,g})(T_4 - T_2) \qquad [\text{R} \mid \text{K}] \qquad (5.29)$$

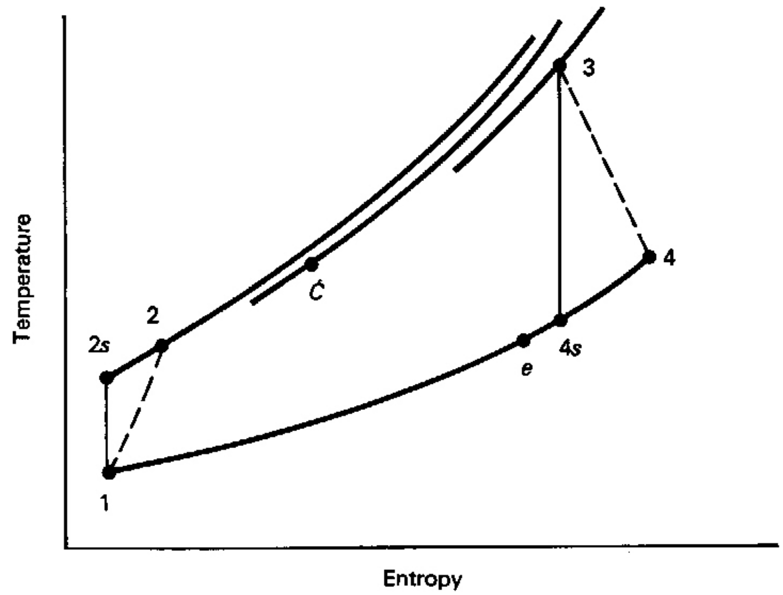

FIGURE 5.7 Notation for Example 5.2.

The regenerator exhaust gas temperature reduction, $T_4 - T_e$, is seen to be jointly proportional to the effectiveness and to the maximum temperature potential, $T_4 - T_2$.

The regenerator, like other heat exchangers, is designed to have minimal pressure losses on both air and gas sides. These may be taken into account by the fractional pressure drop approach discussed in connection with the combustor.

EXAMPLE 5.2

Let's say we are adding a heat exchanger with an effectiveness of 75% to the engine studied in Example 5.1. Assume that the same frictional pressure loss factor applies to both the heat exchanger air-side and combustor as a unit, and that gas-side pressure loss in the heat exchanger is negligible. Evaluate the performance of the modified engine.

Solution

The solution in spreadsheet format, expressed in terms of the notation of Figures 5.6 and 5.7, is shown in Table 5.2. Examination of the spreadsheet and of the T-s diagram in Figure 5.7 shows that the entry and exit states of the turbomachines are not influenced by the addition of the heat exchanger, as expected. (There would have been a slight influence if a different pressure loss model had been assumed.)

With the heat exchanger, it is seen that the combustor inlet temperature has increased about 262° and the exhaust temperature reduced 229°. The net work and

TABLE 5.2 Spreadsheet Solution to Example 5.2

Regenerative cycle gas turbine calculation

	VALUE	EQUATION
air isentropic exponent = k	1.40	given
(k-1)/k = e,a	0.2857	(k-1)/k
compressor inlet temperature = T1,R	520.00	given
compressor efficiency = etac	0.86	given
compressor pressure ratio = r	6.00	given
compressor isentropic exit temp. = T2s,R	867.63	+T1r^e,a
compressor true exit temp.= T2,R	924.22	+T1+(T2s-T1)/etac
compressor work = Wc,Btu/lb	-97.01	0.24(T1-T2)
combustor fractional pressure drop = fpl	0.04	given
turbine pressure ratio = rt	5.76	r(1-fpl)
turbine inlet temp = T3,R	1860.00	given
hot gas isentropic exponent = kg	1.33	given
(kg-1)/kg = e,g	0.25	(kg-1)/kg
hot gas heat capacity = cp,g, Btu/lb-R	0.2744	given
turbine isentropic exit temp. = T4s,R	1200.63	T3/(rt)^e,g
turbine isentropic efficiency = etat	0.89	given
turbine true exit temp. = T4,R	1273.16	T3-etat(T3-T4s)
turbine work = Wt,Btu/lb	161.03	cp,g(T3-T4)
regenerator effectiveness = etareg	0.75	given
combustor inlet temperature = Tc,R	1185.92	T2+etareg(T4-T2)
regenerator gas exit temp. = Te,R	1044.26	T4-0.24(Tc-T2)/cp,g
combustor heat addition = Qa, Btu/lb	184.97	cp,g(T3-Tc)
net work = Wn,Btu/lb	64.02	Wt+Wc
thermal efficiency	0.35	Wn/Qa
work ratio	1.66	Wt/|Wc|

work ratio are clearly unchanged. Most importantly, however, the thermal efficiency has increased 10 percentage counts over the simple cycle case in Example 5.1. Such a gain must be traded off against the added volume, weight, and expense of the regenerator. The efficiency gain and the associated penalties may be acceptable in stationary power and ground and marine transportation applications, but are seldom feasible in aerospace applications. Each case, of course, must be judged on its own merits.

Figure 5.8 shows the influence of regenerator effectiveness and turbine inlet temperature on the performance of the gas turbine, all other conditions being the same as in the example. The values for $\eta_{reg} = 0$ correspond to a gas turbine without regenerator. The abscissa is arbitrarily truncated at $\eta_{reg} = 0.8$ because gas turbine heat exchanger effectivenesses usually do not exceed that value. The impressive influence of both design parameters is a strong motivator for research in heat exchangers and high-temperature materials. The use of regeneration in automotive gas turbines is virtually mandated because good fuel economy is so important.

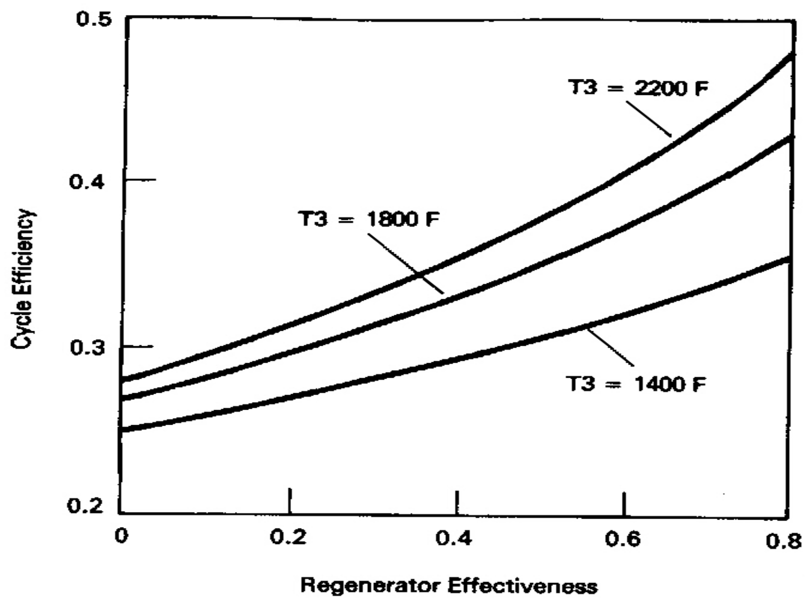

FIGURE 5.8 Effect of regenerator effectiveness on gas turbine thermal efficiency.

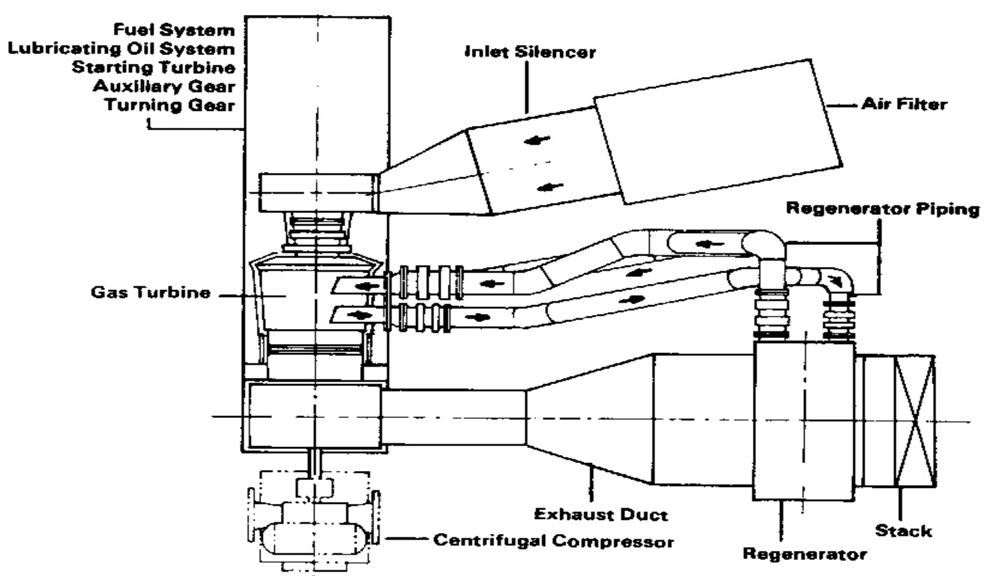

FIGURE 5.9 Layout of a regenerative gas turbine for a pipeline compressor station. (Courtesy of Westinghouse Canada.)

Figure 5.9 shows the layout of a regenerative gas turbine serving a pipeline compressor station. Gas drawn from the pipeline may be used to provide the fuel for remotely located gas-turbine-powered compressor stations. (A later figure, Figure 5.12, shows details of the turbomachinery of this gas turbine.)

5.6 Two-Shaft Gas Turbines

Problems in the design of turbomachinery for gas turbines and in poor part-load or off-design performance are sometimes avoided by employing a *two-shaft gas turbine*, in which the compressor is driven by one turbine and the load by a second turbine. Both shafts may be contained in a single structure, or the turbines may be separately packaged. Figure 5.10 shows the flow and T-s diagrams for such a configuration. The turbine that drives the compressor is called the *compressor turbine*. The compressor, combustor, compressor-turbine combination is called the *gas generator, or gasifier,* because its function is to provide hot, high-pressure gas to drive the second turbine, the *power turbine*. The compressor-turbine is sometimes also referred to as the *gasifier turbine* or *gas-generator turbine*.

The analysis of the two-shaft gas turbine is similar to that of the single shaft machine, except in the determination of the turbine pressure ratios. The pressure rise produced by the compressor must be shared between the two turbines. The manner in which it is shared is determined by a power, or work, condition. The *work condition* expresses mathematically the fact that the work produced by the gasifier turbine is used to drive the compressor alone. As a result, the gas generator turbine pressure ratio, p_3/p_4, is just high enough to satisfy the compressor work requirement.

Thus the compressor power (work) input is the same as the delivered gas-generator turbine power(work) output:

$$|w_c| = \eta_{mech}(1+f)w_t \qquad [\text{Btu/lb}_m \mid \text{kJ/kg}] \qquad (5.30)$$

where f is the fuel–air ratio and η_{mech} is the mechanical efficiency of transmission of power from the turbine to the compressor. The mechanical efficiency is usually close to unity in a well-designed gas turbine. For this reason, it was not included in earlier analyses.

The gasifier turbine work may be written in terms of the turbine pressure ratio:

$$w_t = \eta_t c_{p,g} T_3(1 - T_{4s}/T_3)$$

$$= \eta_t c_{p,g} T_3[1 - 1/(p_3/p_4)^{(kg-1)/kg}] \qquad [\text{Btu/lb}_m \mid \text{kJ/kg}] \qquad (5.31)$$

With the compressor work determined, as before, by the compressor pressure ratio and the isentropic efficiency, the compressor-turbine pressure ratio, p_3/p_4, is obtained by combining Equations (5.30) and (5.31):

$$p_3/p_4 = [1 - |w_c|/\eta_{mech}\eta_t c_{p,g}(1+f)T_3]^{-kg/(kg-1)}$$

$$= (1 - \text{wf})^{-kg/(kg-1)} \qquad [\text{dl}] \qquad (5.32)$$

where wf is the positive, dimensionless work factor, $|w_c|/\eta_{mech}\eta_t c_{p,g}(1+f)T_3$, used as

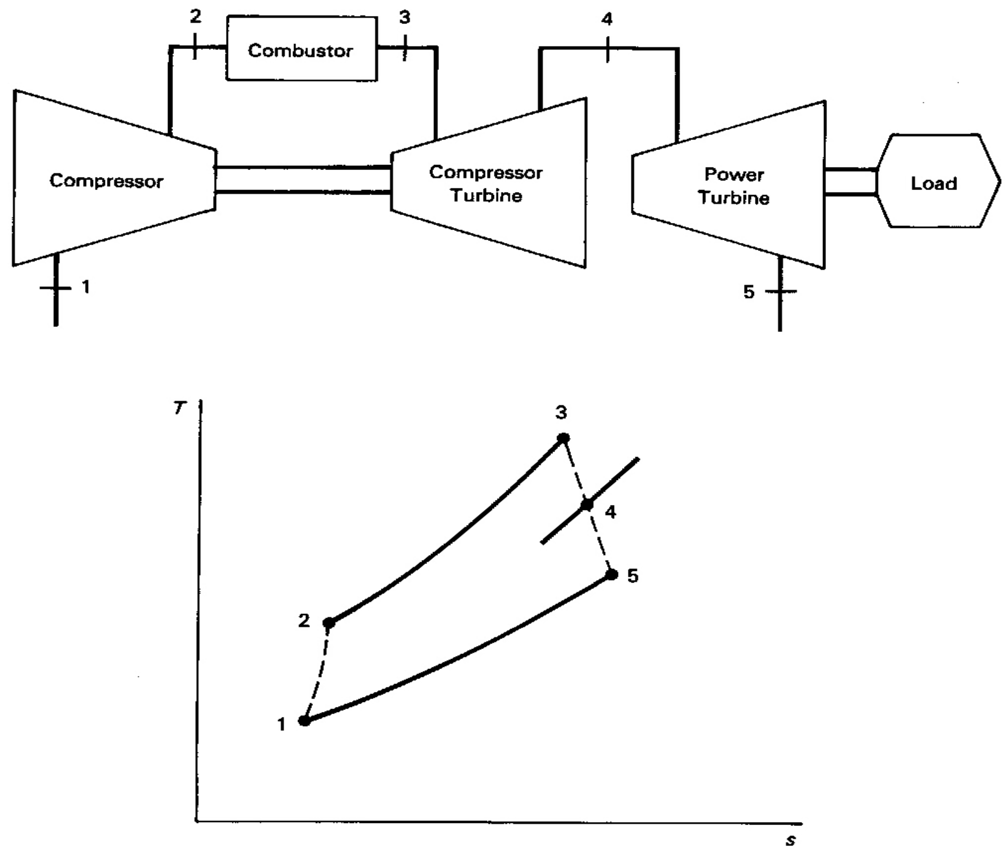

FIGURE 5.10 Two-shaft gas turbine.

a convenient intermediate variable. The power turbine pressure ratio may then be determined from the identity $p_4/p_5 = p_4/p_1 = (p_4/p_3)(p_3/p_2)(p_2/p_1)$. This shows that the power turbine pressure ratio is the compressor pressure ratio divided by the gasifier turbine pressure ratio when there is no combustion chamber pressure loss ($p_3 = p_2$). With the pressure ratios known, all the significant temperatures and performance parameters may be determined.

EXAMPLE 5.3

Let's consider a two-shaft gas turbine with a regenerative air heater. The compressor pressure ratio is 6, and the compressor and gas generator turbine inlet temperatures are 520°R and 1860°R, respectively. The compressor, gasifier turbine, and power turbine isentropic efficiencies are 0.86, 0.89, and 0.89, respectively. The regenerator effectiveness is 75%, and a 4% pressure loss is shared by the high-pressure air side of the regenerator and the combustor. Determine the pressure ratios of the two turbines, and the net work, thermal efficiency, and work ratio of the engine.

TABLE 5.3 Spreadsheet Solution to Example 5.3

Two-shaft regenerative gas turbine calculation		
	VALUE	EQUATION
air isentropic exponent = k	1.40	given
(k-1)/k = e,a	0.2857	(k-1)/k
compressor inlet temperature = T1,R	520.00	given
compressor efficiency = etac	0.86	given
gas gen. turbine efficiency = etatg	0.89	given
power turbine efficiency = etatp	0.89	given
compressor pressure ratio = r	6.00	given
compressor isentropic exit temp. = T2s,R	867.63	+T1r^e,a
compressor true exit temp.= T2,R	924.22	+T1+(T2s-T1)/etac
compressor work = Wc,Btu/lb	-97.01	0.24(T1-T2)
combustor fractional pressure drop = fpl	0.04	given
available turbine pressure ratio = p3/p1	5.76	r(1-fpl)
turbine inlet temp = T3,R	1860.00	given
hot gas isentropic exponent = kg	1.33	given
(kg-1)/kg = e,g	0.25	(kg-1)/kg
hot gas heat capacity = cp,g, Btu/lb-R	0.2744	given
work factor (intermediate var.) = wf	-0.21	Wc/(etatg*cp,g*T3)
gas gen. turbine pressure ratio = rtg	2.61	1/[1+wf]^(1/e,g)
power turbine pressure ratio = rtp	2.20	r(1-fpl)/rtg
GG turbine isen. exit temp. = T4s,R	1462.76	T3/(rtg)^e,g
GG turbine true exit temp. = T4,R	1506.46	T3-etatg(T3-T4s)
Power turbine isen. exit temp. = T5s,R	1236.49	T4/(rtp)^e,g
Power turbine true exit temp. = T5,R	1266.19	T4-etatp(T4-T5s)
Power turbine work = Wt,Btu/lb	65.93	cp,g(T4-T5)
regenerator effectiveness = etareg	0.75	given
combustor inlet temperature = Tc,R	1180.69	T2+etareg(T5-T2)
regenerator gas exit temp. = Te,R	1041.86	T5-0.24(Tc-T2)/cp,g
combustor heat addition = Qa, Btu/lb	186.40	cp,g(T3-Tc)
net work = Wn,Btu/lb	65.93	Wt
thermal efficiency	0.35	Wn/Qa
work ratio	1.68	1+Wt/|Wc|

Solution

The solution in spreadsheet form shown in Table 5.3 follows the notation of Figure 5.10. The solution proceeds as before, until the calculation of the turbine pressure ratios. The available pressure ratio shared by the two turbines is $p_3/p_5 = p_3/p_1 = (p_2/p_1)(p_3/p_2) = r\,(1 - \text{fpl}) = 5.76$. The gasifier turbine pressure ratio is determined by the work-matching requirement of the compressor and its driving turbine, as expressed in Equation (5.32), using the dimensionless compressor work factor, wf. The resulting gas generator and power turbine pressure ratios are 2.61 and 2.2, respectively.

Comparison shows that the design point performance of the two-shaft gas turbine studied here is not significantly different from that of the single-shaft machine considered an Example 5.2. While the performance of the two machines is found to be essentially the same, the single-shaft machine is sometimes preferred in applications

TABLE 5.4 Spreadsheet 5-4.WK1

Two-shaft regenerative gas turbine comparison

k	1.40	1.40	1.40	1.40	1.40	1.40
(k-1)/k = e,a	0.2857	0.2857	0.2857	0.2857	0.2857	0.2857
T1,R	520.00	520.00	520.00	520.00	520.00	520.00
etac	0.86	0.86	0.86	0.86	0.86	0.86
etatg	0.89	0.89	0.89	0.89	0.89	0.89
etatp	0.89	0.89	0.89	0.89	0.89	0.89
r	2.00	3.00	4.00	5.00	6.00	7.00
T2s,R	633.89	711.74	772.72	823.59	867.63	906.69
T2,R	652.43	742.96	813.86	873.01	924.22	969.64
Wc,Btu/lb	-31.78	-53.51	-70.53	-84.72	-97.01	-107.91
fpl	0.04	0.04	0.04	0.04	0.04	0.04
p3/p1	1.92	2.88	3.84	4.80	5.76	6.72
T3,R	1860.00	1860.00	1860.00	1860.00	1860.00	1860.00
kg	1.33	1.33	1.33	1.33	1.33	1.33
(kg-1)/kg=e,g	0.25	0.25	0.25	0.25	0.25	0.25
cp,g,Btu/lb-R	0.2744	0.2744	0.2744	0.2744	0.2744	0.2744
work factor	-0.07	-0.12	-0.16	-0.19	-0.21	-0.24
rtg	1.34	1.65	1.96	2.28	2.61	2.96
rtp	1.44	1.74	1.96	2.10	2.20	2.27
T4s,R	1729.86	1640.89	1571.22	1513.09	1462.76	1418.12
T4,R	1744.17	1664.99	1602.98	1551.25	1506.46	1466.73
T5s,R	1593.19	1448.76	1355.57	1288.31	1236.49	1194.83
T5,R	1609.80	1472.55	1382.79	1317.23	1266.19	1224.74
Wt,Btu/lb	36.87	52.81	60.42	64.21	65.93	66.40
etareg	0.75	0.75	0.75	0.75	0.75	0.75
Tc,R	1370.45	1290.15	1240.56	1206.18	1180.69	1160.97
Te,R	981.78	993.95	1009.58	1025.83	1041.86	1057.40
Qa, Btu/lb	134.33	156.37	169.98	179.41	186.40	191.81
Wn,Btu/lb	36.87	52.81	60.42	64.21	65.93	66.40
thermal eff.	0.27	0.34	0.36	0.36	0.35	0.35
work ratio	2.16	1.99	1.86	1.76	1.68	1.62

with fixed operating conditions where good part-load performance over a range of speeds is not important. On the other hand, the independence of the speeds of the gas generator and power turbine in the two-shaft engine allows acceptable performance over a wider range of operating conditions.

Let us examine further the characteristics of regenerative two-shaft gas turbines, starting with the spreadsheet reproduced in Table 5.3. By copying the value column of that spreadsheet to several columns to the right, a family of calculations with identical methodologies may be performed.. The spreadsheet /EDIT-FILL command may then be used to vary a parameter in a given row by creating a sequence of numbers with a specified starting value and interval. Such a parametric study of the influence of compressor pressure ratio on two-shaft regenerative gas turbine performance is shown in Table 5.4, where the pressure ratio is varied from 2 to 7. The fifth numeric column contains the values from Table 5.3. The data of Table 5.4 are included in the Example

5-4.wk3 spreadsheet that accompanies this text.

Table 5.4 shows that, for the given turbine inlet temperature, the thermal efficiency maximum is at a pressure ratio between 4 and 5, while the net work maximum is at a pressure ratio of about 7. The work ratio is continually declining because the magnitude of the compressor work requirement grows faster with compressor pressure ratio than the turbine work does.

Figure 5.11, plotted using the spreadsheet, compares the performance of the regenerative two-shaft gas turbine with a nonregenerative two-shaft engine ($\eta_{reg} = 0$). Net work for both machines has the same variation with pressure ratio. But notice the high efficiency attained with a low-compressor pressure ratio, a significant advantage attributable to regeneration.

A cutaway view of a two-shaft regenerative gas turbine of the type used in pipeline compressor stations such as that shown in Figure 5.9 is seen in Figure 5.12. The figure shows that the compressor blade heights decrease in the direction of flow as the gas is compressed. The exhaust from the last of the sixteen compressor stages is reduced in velocity by a diffusing passage and then exits through the right window-like flange, which connects to a duct (not shown) leading to the regenerator. The heated air from the external regenerator reenters the machine combustor casing, where it flows around and into the combustor cans, cooling them. The air entering near the combustor fuel nozzles mixes with the fuel and burns locally in a near-stoichiometric mixture. As the mixture flows downstream, additional secondary air entering the combustor through slots in its sides mixes with, and reduces the temperature of, the combustion gas before it arrives at the turbine inlet.

A cutaway view of an industrial two-shaft gas turbine with dual regenerators is presented in Figure 5.13. From the left, the air inlet and radial compressor and axial flow gasifier turbine and power turbine are seen on the axis of the machine, with the combustion chamber above and one of the rotary regenerators at the right. Due to the relatively low pressure ratios required by regenerative cycles, centrifugal compressors are normally used in regenerative machines because of their simplicity, good efficiency, compactness, and ruggedness.

Performance data for the turbine of Figure 5.13 is graphed in Figure 5.14. The GT 404 gas turbine delivers about 360 brake horsepower at 2880-rpm output shaft speed. The torque-speed curve of Figure 5.14 shows an important characteristic of two-shaft gas turbines with respect to off-design point operation. Whereas the compressor pressure ratio and output torque of a single-shaft gas turbine drop as the shaft speed drops, the compressor speed and pressure ratio in a two-shaft machine is independent of the output speed. Thus, as the output shaft speed changes, the compressor may maintain its design speed and continue to develop high pressure and mass flow. Thus the torque at full stall of the output shaft of the GT404 is more than twice the full-load design torque. This high stall torque is superior to that of reciprocating engines and is important in starting and accelerating rotating equipment that has high initial turning resistance. This kind of engine may be used in truck, bus, and marine applications as well as in an industrial setting.

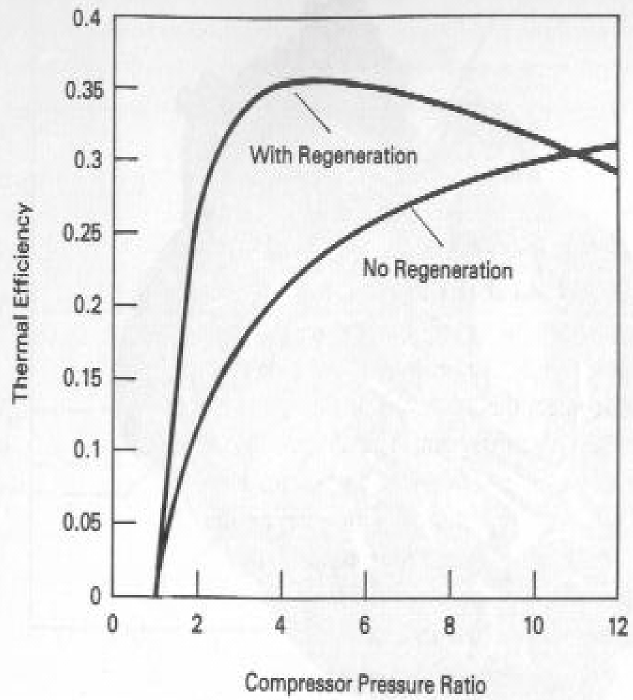

FIGURE 5.11 Performance of two-shaft gas turbines.

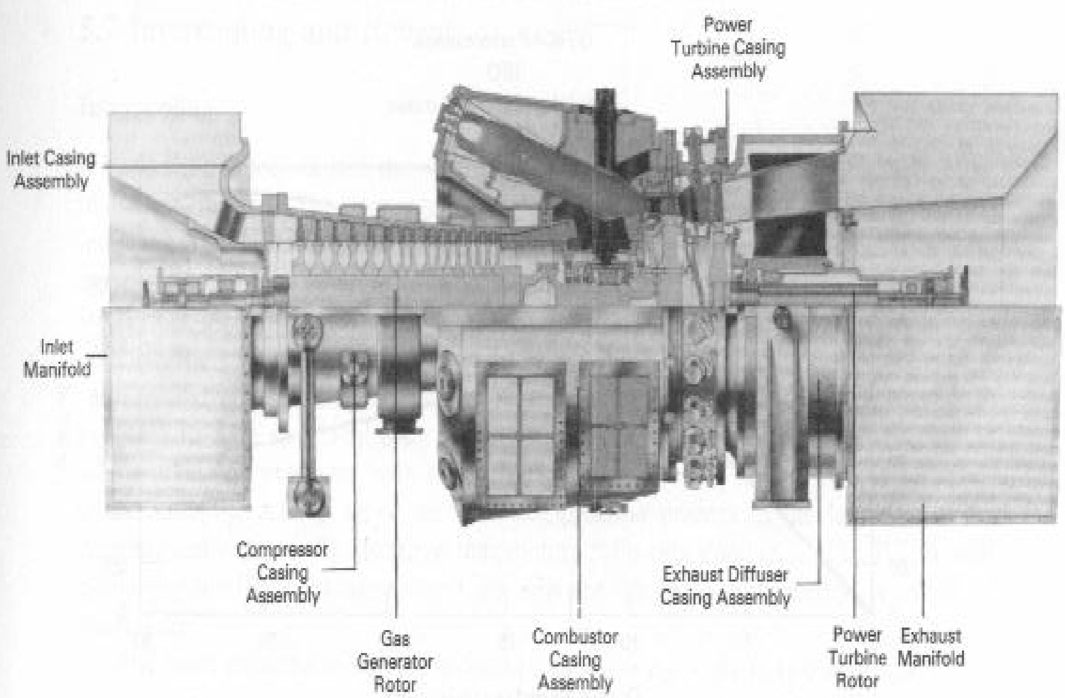

FIGURE 5.12 Two-shaft regenerative combustion turbine. (Courtesy of Westinghouse Canada.)

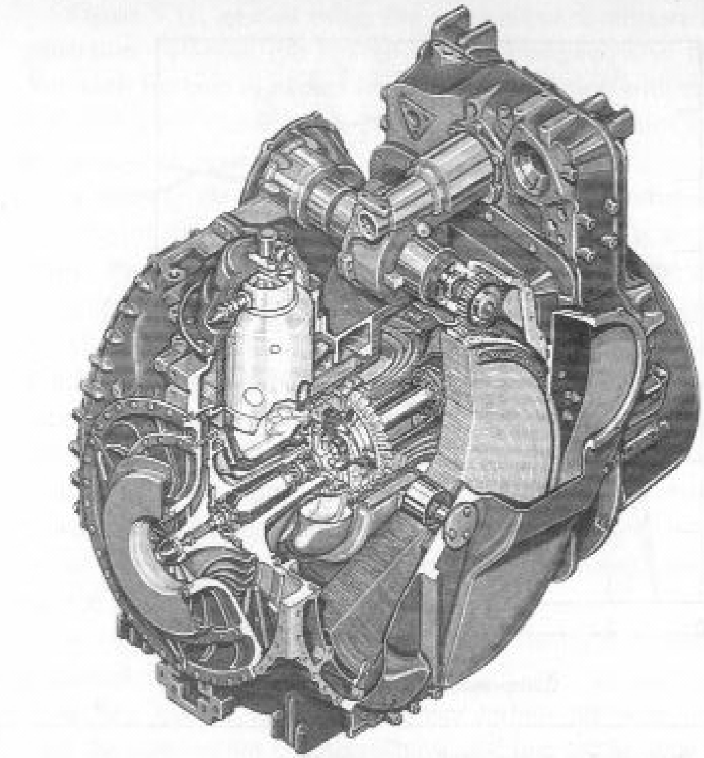

FIGURE 5.13 GT404 regenerative industrial two-shaft gas turbine. (Courtesy of the Allison Gas Turbine Division, General Motors Corporation.)

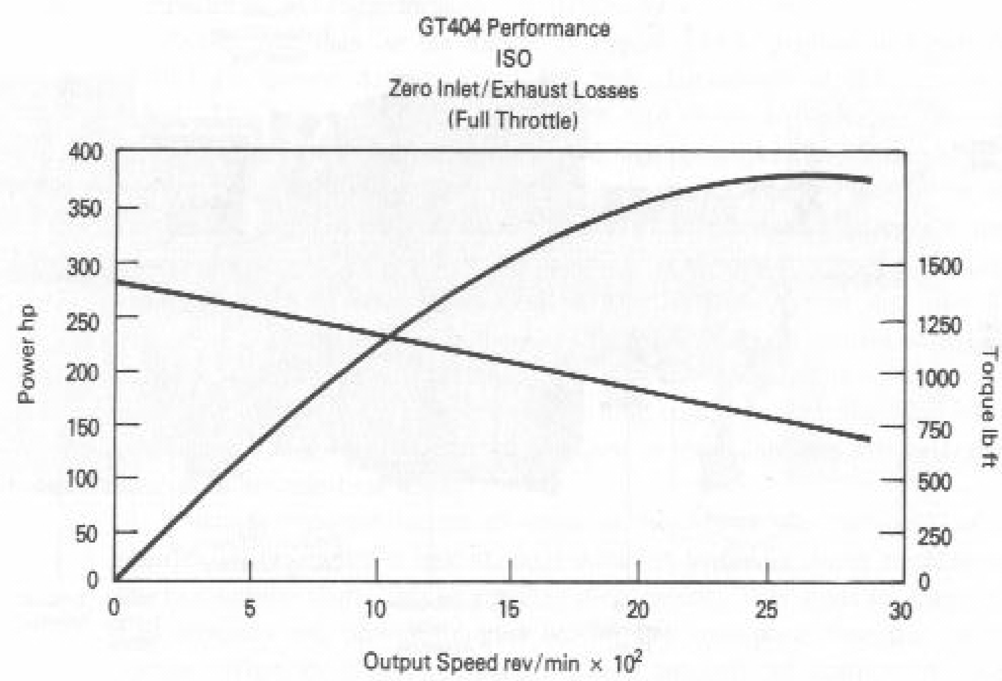

FIGURE 5.14 Torque and power characteristics of the GT404 gas turbine. (Courtesy of the Allison Gas Turbine Division, General Motors Corporation.)

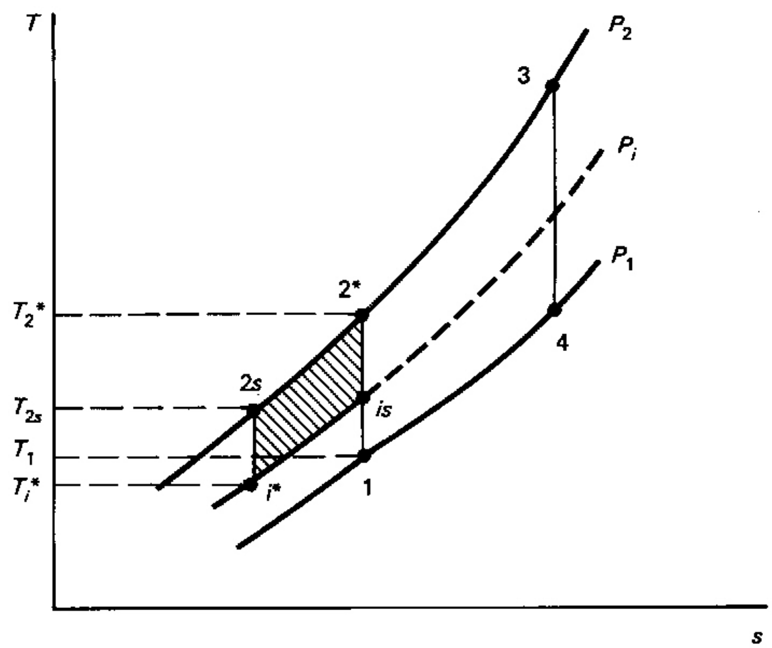

FIGURE 5.15 Two-stage compression with intercooling.

A unique patented feature of some of the Allison gas turbines, called "power transfer," is the ability to link the duel shafts. A hydraulic clutch mechanism between the two turbine shafts acts to equalize their speeds. This tends to improve part-load fuel economy, and provides engine braking and overspeed protection for the power turbine. When the clutch mechanism is fully engaged, the shafts rotate together as a single-shaft machine.

5.7 Intercooling and Reheat

Intercooling

It has been pointed out that the work of compression extracts a high toll on the output of the gas turbine. The convergence of lines of constant pressure on a T-s diagram indicates that compression at low temperatures reduces compression work. The ideal compression process would occur isothermally at the lowest available temperature. Isothermal compression is difficult to execute in practice. The use of multistage compression with intercooling is a move in that direction.

Consider replacing the isentropic single-stage compression from p_1 to $p_2 = p_{2*}$ in Figure 5.15 with two isentropic stages from p_1 to p_{is} and p_{i*} to p_{2s}. Separation of the compression processes with a heat exchanger that cools the air at T_{is} to a lower temperature T_{i*} acts to move the final compression process to the left on the T-s diagram and reduces the discharge temperature following compression to T_{2s}. A heat exchanger used to cool compressed gas between stages of compression is called an *intercooler*.

The work required to compress from p_1 to $p_{2s} = p_{2*} = p_2$ in two stages is

$$w_c = c_p\,[(T_1 - T_{is}) + (T_{1*} - T_{2s})] \qquad [\text{Btu/lb}_m \mid \text{kJ/kg}]$$

Note that intercooling increases the net work of the reversible cycle by the area *is–i*–2s–2*–is*. The reduction in the work due to two-stage intercooled compression is also given by this area. Thus intercooling may be used to reduce the work of compression between two given pressures in any application. However, the favorable effect on compressor work reduction due to intercooling in the gas turbine application may be offset by the obvious increase in combustor heat addition, $c_p\,(T_{2*} - T_{2s})$, and by increased cost of compression system. The next example considers the selection of the optimum pressure level for intercooling, $p_i = p_{is} = p_{i*}$.

EXAMPLE 5.4

Express the compressor work, for two-stage compression with intercooling back to the original inlet temperature, in terms of compressor efficiencies and pressure ratios. Develop relations for the compressor pressure ratios that minimize the total work of compression in terms of the overall pressure ratio.

Solution

Taking $T_{i*} = T_1$ as directed in the problem statement, and letting $r = p_2/p_1$, $r_1 = p_{is}/p_1$ and $r_2 = p_{2s}/p_{i*} = r/r_1$ as in Figure 5.15, we get for the compression work,

$$w_c = c_p\,[(T_1 - T_{is})/\eta_{c1} + (T_1 - T_{2s})/\eta_{c2}]$$

$$= c_p\,T_1[(1 - T_{is}/T_1)/\eta_{c1} + (1 - T_{2s}/T_1)/\eta_{c2}]$$

$$= c_p\,T_1[(1 - r_1^{(k-1)/k})/\eta_{c1} + (1 - r_2^{(k-1)/k})/\eta_{c2}] \qquad [\text{Btu/lb}_m \mid \text{kJ/kg}]$$

Eliminating r$_2$, using $r = r_1 r_2$, yields

$$w_c = c_p\,T_1[(1 - r_1^{(k-1)/k})/\eta_{c1} + (1 - (r/r_1)^{(k-1)/k})/\eta_{c2}]$$

Differentiating with respect to r_1 for a fixed r and setting the result equal to zero, we obtain

$$-r_1^{-1/k}/\eta_{c1} + (r^{(k-1)/k}/\eta_{c2})\,r_1^{-(2k-1)/k} = 0$$

which simplifies to

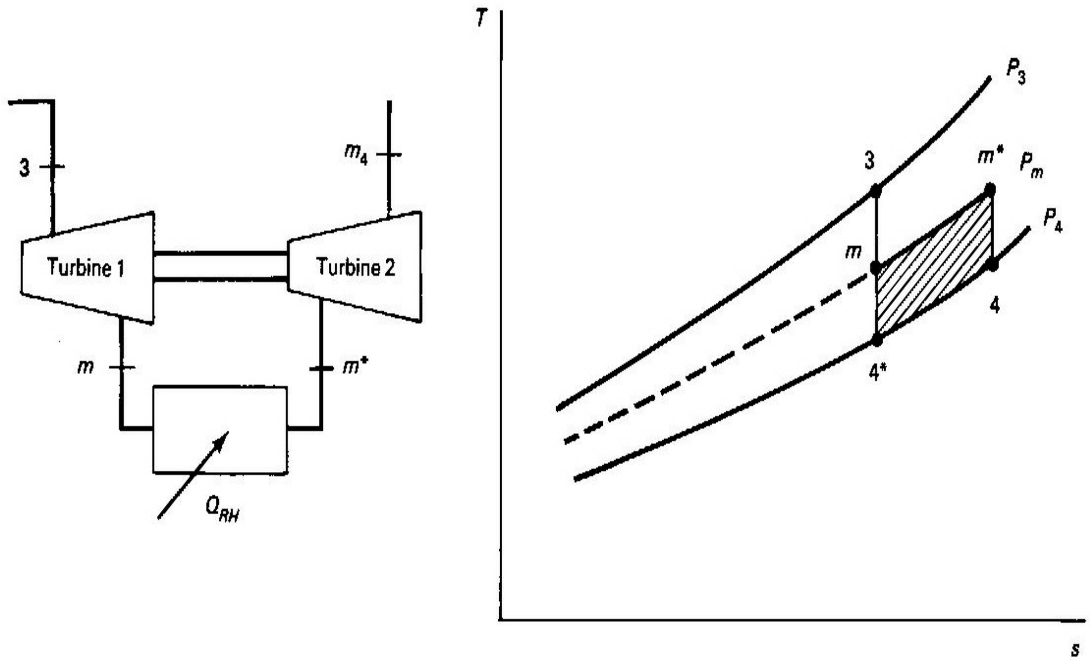

FIGURE 5.16 Gas turbine reheat.

$$r_{1opt} = (\eta_{c1}/\eta_{c2})^{k/(2k-1)}\, r^{1/2}$$

Using this result we find also that

$$r_{2opt} = (\eta_{c2}/\eta_{c1})^{k/(2k-1)}\, r^{1/2}$$

Examination of these equations shows that, for compressors with equal efficiencies, both compressor stages have the same pressure ratio, which is given by the square root of the overall pressure ratio. For unequal compressor efficiencies, the compressor with the higher efficiency should have the higher pressure ratio.

Reheat

Let us now consider an improvement at the high-temperature end of the cycle. Figure 5.16 shows the replacement of a single turbine by two turbines in series, each with appropriately lower pressure ratios, and separated by a reheater. The *reheater* may be a combustion chamber in which the excess oxygen in the combustion gas leaving the first turbine burns additional fuel, or it may be a heater in which external combustion provides the heat necessary to raise the temperature of the working fluid to T_{m*}. The high temperature at the low-pressure turbine inlet has the effect of increasing the area of the cycle by m–m*–4–4*m and hence of increasing the net work.

Like intercooling, the increase in net work is made possible by the spreading of the constant pressure lines on the T-s diagram as entropy increases. Thus the *increase* in turbine work is

$$\Delta w_t = c_{p,g}[(T_{m*} - T_4) - (T_m - T_{4*})] \qquad [\text{Btu/lb}_m \mid \text{kJ/kg}] \qquad (5.33)$$

Also as with intercooling, the favorable effect in increasing net work is offset by the reduction of cycle efficiency resulting from increased addition of external heat from the reheater:

$$q_{rh} = c_{p,g}(T_{m*} - T_m) \qquad [\text{Btu/lb}_m \mid \text{kJ/kg}] \qquad (5.34)$$

As with intercooling, the question arises as to how the intermediate pressure for reheat will be selected. An analysis similar to that of Example 5.4 shows the unsurprising result that the reheat pressure level should be selected so that both turbines have the same expansion ratio if they have the same efficiencies and the same inlet temperatures.

Combining Intercooling, Reheat, and Regeneration

Because of their unfavorable effects on thermal efficiency, intercooling and reheat alone or in combination are unlikely to be found in a gas turbine without another feature that has already been shown to have a favorable influence on gas turbine fuel economy: a regenerator. The recuperator or regenerator turns disadvantage into advantage in a cycle involving intercooling and/or reheat. Consider the cycle of Figure 5.17, which incorporates all three features.

The increased turbine discharge temperature T_4 produced by reheat and the decreased compressor exit temperature T_2 due to intercooling both provide an enlarged temperature potential for regenerative heat transfer. Thus the heat transfer $c_p(T_c - T_2)$ is accomplished by an internal transfer of heat from low pressure turbine exhaust gas. This also has the favorable effect of reducing the temperature of the gas discharged to the atmosphere. The requisite external heat addition for this engine is then

$$q_a = c_p[(T_3 - T_c) + (T_{m*} - T_m)] \qquad [\text{Btu/lb}_m \mid \text{kJ/kg}] \qquad (5.35)$$

Thus the combination of intercooling, reheat, and regeneration has the net effect of raising the average temperature of heat addition and lowering the average temperature of heat rejection, as prescribed by Carnot for an efficient heat engine.

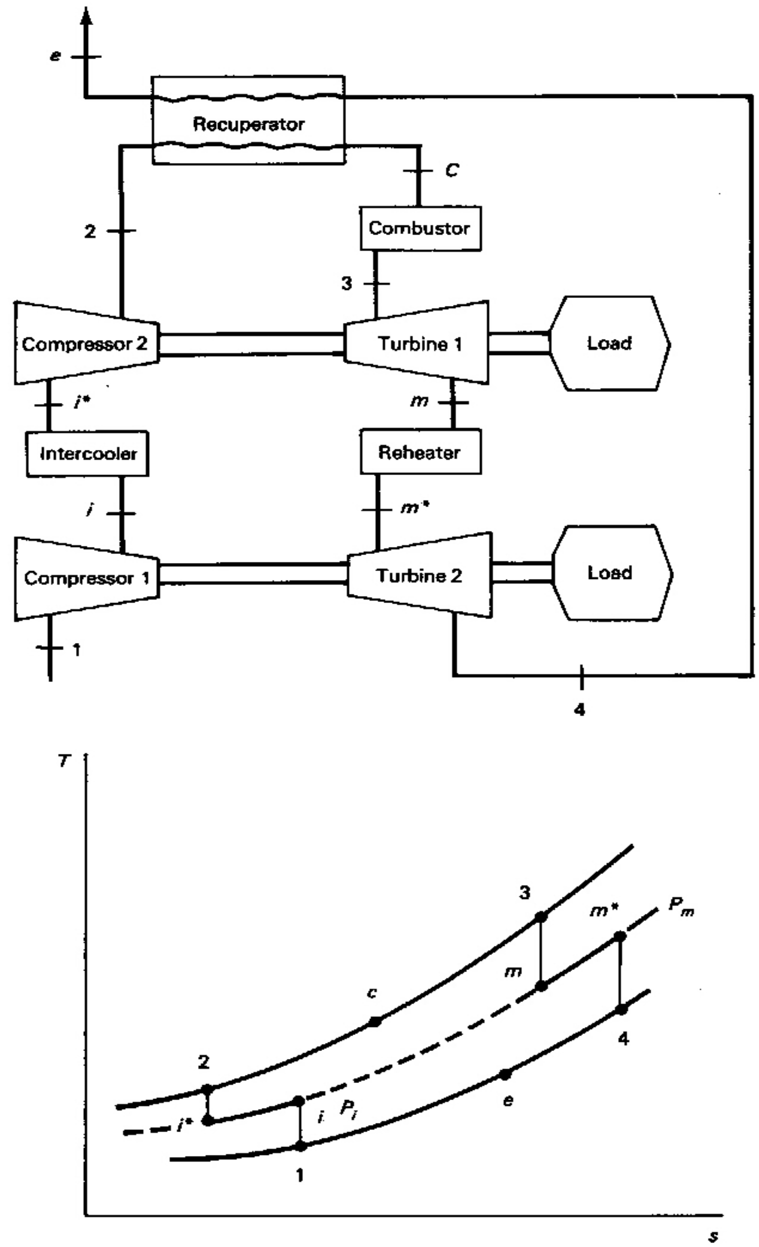

FIGURE 5.17 Gas turbine with intercooling, reheat, and regeneration.

The Ericsson cycle

Increasing the number of intercoolers and reheaters without changing the overall pressure ratio may be seen to cause both the overall compression and the overall expansion to approach isothermal processes. The resulting reversible limiting cycle, consisting of two isotherms and two isobars, is called the Ericsson cycle. With perfect internal heat

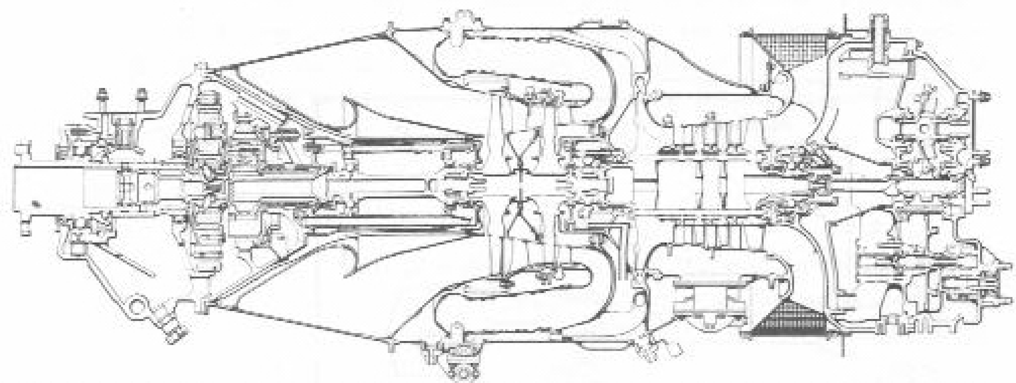

FIGURE 5.18 PT-6 turboprop engine cross-section. (Courtesy of Pratt and Whitney Canada.)

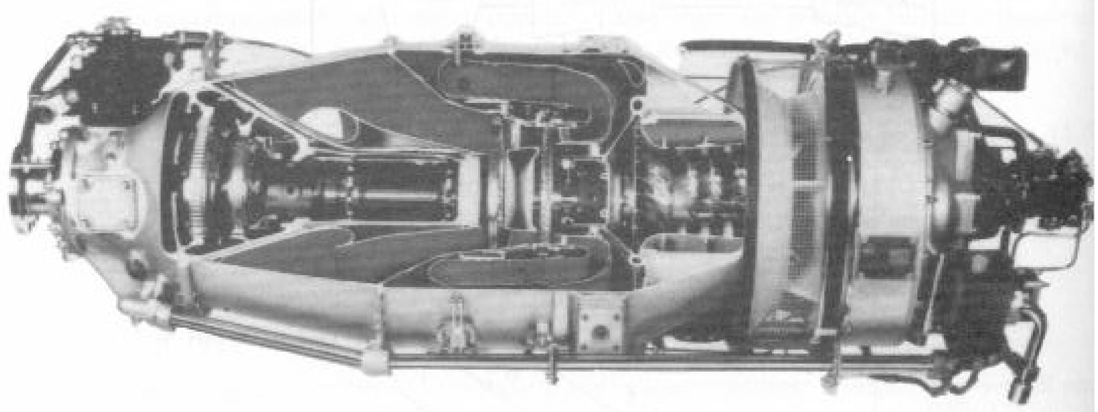

FIGURE 5.19 Cutaway of the PT-6 turboprop engine. (Courtesy of Pratt and Whitney Canada.)

transfer between isobaric processes, all external heat addition would be at the maximum temperature of the cycle and all heat rejected at the lowest temperature. Analysis of the limiting reversible cycle reveals, as one might expect, that its efficiency is that of the Carnot cycle. Plants with multistage compression, reheat, and regeneration can have high efficiencies; but complexity and high capital costs have resulted in few plants that actually incorporate all these features.

5.8 Gas Turbines in Aircraft –Jet engines

Gas turbines are used in aircraft to produce shaft power and hot, high-pressure gas for jet propulsion. Turbine shaft power is used in turboprop aircraft and helicopters to drive propellers and rotors. A modern turboprop engine and an aircraft that uses it are shown in Figures 5.18 through 5.20. While its jet exhaust provides some thrust, the bulk of the propulsive thrust of the turboprop is provided by its propeller. The rear-

FIGURE 5.20 First production Beech Starship in flight. (Photo courtesy of Beech Aircraft Corp.)

ward acceleration of a large air mass by the propeller is responsible for the good fuel economy of turboprop aircraft. Thus the turboprop is popular as a power plant for small business aircraft. At higher subsonic flight speeds, the conventional propeller loses efficiency and the turbojet becomes superior.

Auxiliary power units, APUs, are compact gas turbines that provide mechanical power to generate electricity in transport aircraft while on the ground. The thermodynamic fundamentals of these shaft-power devices are the same those of stationary gas turbines, discussed earlier. Their design, however, places a premium on low weight and volume and conformance to other constraints associated with airborne equipment. Thus their configuration and appearance may differ substantially from those of other stationary gas turbines.

The jet engine consists of a gas turbine that produces hot, high-pressure gas but has zero net shaft output. It is a gasifier. A *nozzle* converts the thermal energy of the hot, high-pressure gas produced by the turbine into a high-kinetic-energy exhaust stream. The high momentum and high exit pressure of the exhaust stream result in a forward thrust on the engine.

Although the analysis of the jet engine is similar to that of the gas turbine, the configuration and design of jet engines differ significantly from those of most stationary gas turbines. The criteria of light weight and small volume, mentioned earlier, apply here as well. To this we can add the necessity of small frontal area to minimize the aerodynamic drag of the engine, the importance of admitting air into the

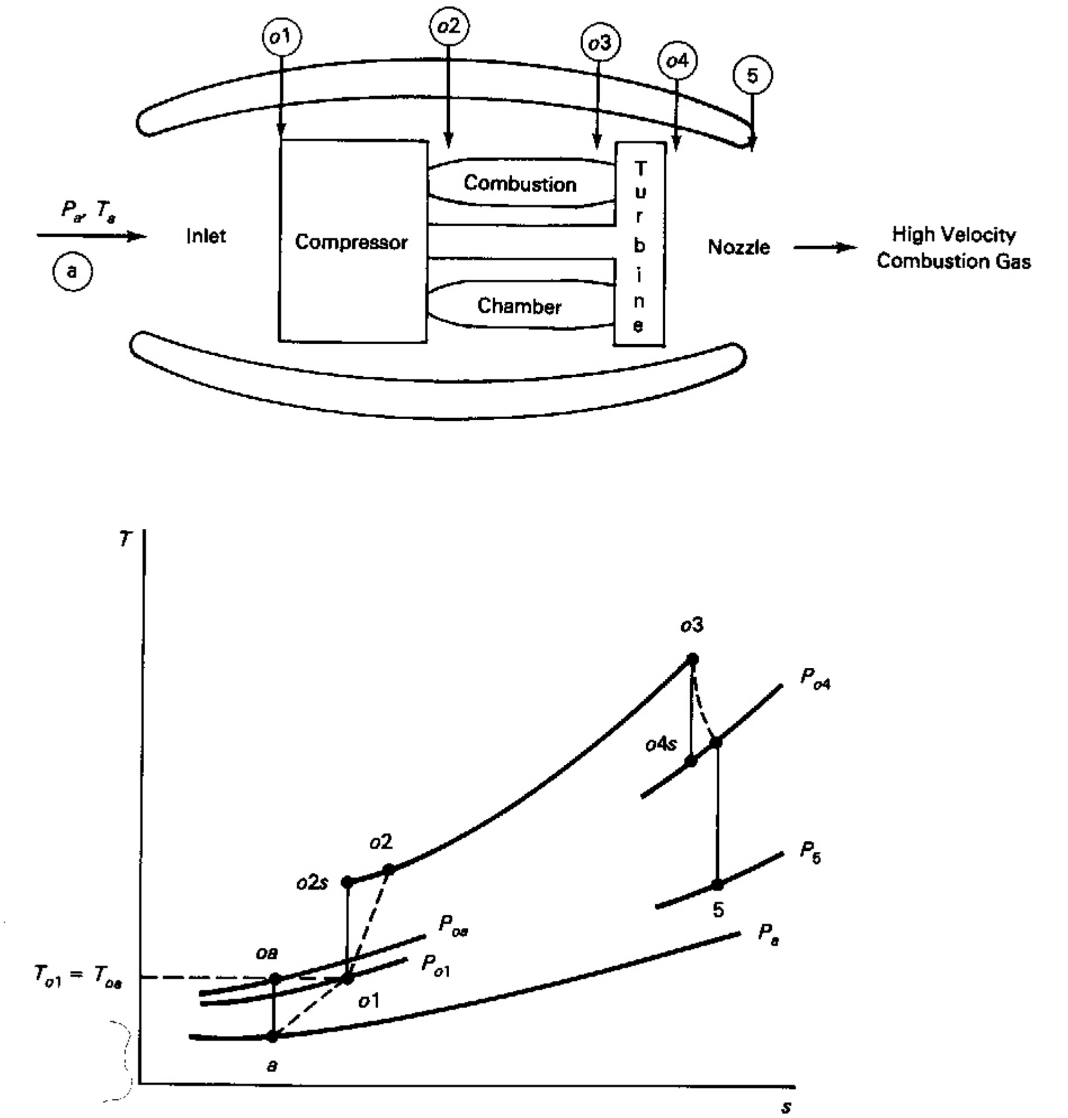

FIGURE 5.21 Jet engine notation and temperature–entropy diagram.

engine as efficiently (with as little stagnation pressure loss) as possible, and the efficient conversion of high-temperature turbine exit gas to a high-velocity nozzle exhaust. The resulting configuration is shown schematically in Figure 5.21.

Up to now we have not been concerned with kinetic energy in the flows in gas turbines, because the flows at the stations of interest are usually designed to have low velocities. In the jet engine, however, high kinetic energy is present in the free stream ahead of the engine and in the nozzle exit flow. The analysis here will therefore be presented in terms of *stagnation*, or *total*, temperatures and pressures, where kinetic energy is taken into account implicitly, as discussed in Section 1.7. The preceding analyses may be readily adapted to deal with the stagnation properties associated with compressible flow. In the following discussion, engine processes are first described and then analyzed.

It should be recalled that if there are no losses, as in an isentropic flow, the stagnation pressure of a flow *remains constant*. All loss mechanisms, such as fluid friction, turbulence, and flow separation, *decrease* stagnation pressure. Only by doing work on the flow (with a compressor, for example) is it possible to *increase* stagnation pressure.

In Figure 5.21, free-stream ambient air, denoted by subscript a, enters an engine inlet that is carefully designed to efficiently decelerate the air captured by its frontal area to a speed low enough to enter the compressor, at station 1, with minimal aerodynamic loss. There is stagnation pressure loss in the inlet, but efficient deceleration of the flow produces static and stagnation pressures at the compressor entrance well above the ambient free-stream static pressure. This conversion of relative kinetic energy of ambient air to increased pressure and temperature in the engine inlet is sometimes called *ram effect*.

The compressor raises the stagnation pressure of the air further to its maximum value at station 2, using power delivered by the turbine. Fuel enters the combustion chamber and is burned with much excess air to produce the high turbine inlet temperature at station 3. We adopt here, for simplicity, the familiar idealization that no pressure losses occur in the combustion chamber. The hot gases then expand through the turbine and deliver just enough power to drive the compressor (the work condition again). The gases leave the turbine exit at station 4, still hot and at a stagnation pressure well above the ambient. These gases then expand through a nozzle that converts the excess pressure and thermal energy into a high-kinetic-energy jet at station 5. The forward thrust on the engine, according to Newton's Second Law, is produced by the reaction to the internal forces that accelerate the internal flow rearward to a high jet velocity and the excess of the nozzle exit plane pressure over the upstream ambient pressure.

Inlet Analysis

Given the flight speed, V_a, and the free-stream static temperature and pressure, T_a and p_a, at a given altitude, the *free-stream stagnation temperature and pressure* are

$$T_{oa} = T_a + V_a^2 / 2c_p \qquad [\mathrm{R} \mid \mathrm{K}] \qquad (5.36)$$

and

$$p_{oa} = p_a(T_{oa}/T_a)^{k/(k-1)} \qquad [\mathrm{lb_f/ft^2} \mid \mathrm{kPa}] \qquad (5.37)$$

Applying the steady-flow energy equation to the streamtube entering the inlet, we find that the stagnation enthalpy $h_{oa} = h_{o1}$ for adiabatic flow. For subsonic flight and supersonic flight at Mach numbers near one, the heat capacity of the air is essentially constant. Thus constancy of the stagnation enthalpy implies constancy of the stagnation temperature. Hence, using Equation (5.36),

$$T_{o1} = T_{oa} = T_a + V_a^2 / 2c_p \qquad [R \mid K] \qquad (5.38)$$

The effects of friction, turbulence, and other irreversibilities in the inlet flow are represented by the *inlet pressure recovery*, PR, defined as

$$\text{PR} = p_{o1} / p_{oa} \qquad [dl] \qquad (5.39)$$

where an isentropic flow through the inlet has a pressure recovery of 1.0. Lower values indicate reduced inlet efficiency and greater losses. For subsonic flow, values on the order of 0.9 to 0.98 are typical. At supersonic speeds the pressure recovery decreases with increasing Mach number.

Compressor Analysis

With the stagnation conditions known at station 1 in Figure 5.21, the compressor pressure ratio, $r = p_{o2} / p_{o1}$, now yields p_{o2}; and the isentropic relation, Equation (1.19), gives the isentropic temperaure, T_{o2s}:

$$T_{o2s} = T_{o1} r^{(k-1)/k} \qquad [R \mid K] \qquad (5.40)$$

The actual compressor discharge stagnation temperature is then obtained from the definition of the compressor efficiency in terms of stagnation temperatures:

$$\eta_{comp} = (T_{o1} - T_{o2s}) / (T_{o1} - T_{o2}) \qquad [dl] \qquad (5.41)$$

Combustor and Turbine Analysis

The turbine inlet temperature, T_{03}, is usually assigned based on turbine blade material considerations. For preliminary analysis it may be assumed that there are negligible pressure losses in the combustion chamber, so that $p_{03} = p_{02}$. As with the two-shaft gas turbine, the condition that the power absorbed by the compressor equal the power delivered by the turbine determines the turbine exit temperature, T_{04}:

$$c_p(T_{o2} - T_{o1}) = (1 + f)c_{p,g} T_{o3} (1 - T_{o4} / T_{o3}) \qquad [Btu \mid kJ] \qquad (5.42)$$

where f is the engine fuel-air ratio, which often may be neglected with respect to 1 (as in our earlier studies) when high precision is not required.

The turbine efficiency equation then yields the isentropic discharge temperature T_{04s}, and Equation (1.19) yields the turbine pressure ratio:

$$T_{o4s} = T_{o3} - (T_{o3} - T_{o4}) / \eta_{turb} \qquad [R \mid K] \qquad (5.43)$$

$$p_{o3} / p_{o4} = (T_{o3}/T_{o4s})^{kg/(kg-1)} \qquad [dl] \qquad (5.44)$$

Thus the stagnation pressure and temperature at station 4 are known. Note that the turbine pressure ratio is usually significantly lower than the compressor pressure ratio.

Nozzle Analysis

The flow is then accelerated to the jet velocity at station 5 by a *convergent nozzle* that contracts the flow area. A well-designed nozzle operating at its design condition has only small stagnation pressure losses. Hence the nozzle here is assumed to be loss-free and therefore isentropic.

Under most flight conditions the exhaust nozzle is *choked*; that is, it is passing the maximum flow possible for its upstream conditions. A choked nozzle has the local flow velocity at its minimum area, or *throat*, equal to the local speed of sound. As a result, simple relations exist between the upstream stagnation conditions at station 4 and the choked conditions at the throat. Thus, for a choked isentropic nozzle,

$$T_{o4} = T_{o5} = T_5 + V_5^{\,2} / 2c_{p,g}$$

$$= T_5 + k_g R T_5 / 2c_{p,g} = T_5 (1 + k_g R / 2c_{p,g})$$

$$= T_5(k_g + 1)/2 \qquad [R \mid K] \qquad (5.45)$$

where $k_g R/c_{p,g} = k_g - 1$. With $k_g = 1.333$ for the combustion gas, this determines the exit temperature T_5. Combining the isentropic relation with Equation (5.45) then gives the nozzle exit static pressure p_5:

$$p_5 / p_{o4} = (T_5 / T_{04})^{kg/(kg-1)}$$

$$= [2/(k_g + 1)]^{kg/(kg-1)} \qquad [dl] \qquad (5.46)$$

EXAMPLE 5.5

The stagnation temperature and pressure leaving a turbine and entering a convergent nozzle are 970.2K. and 2.226 bar, respectively. What is the static pressure and temperature downstream if the nozzle is choked? If the free-stream ambient pressure is 0.54 bar, is the nozzle flow choked? Compare the existing nozzle pressure ratio with the critical pressure ratio.

Solution

If the nozzle is choked, then from Equations (5.45) and (5.46),

$$T_5 = 2T_{o4}/(k_g + 1) = 2(970.2)/2.333 = 831.6 \text{ K}$$

and the static pressure at the nozzle throat is

$$p_5 = p_{o4}[2/(k_g + 1)]^{kg/(kg-1)} = 2.226(2 / 2.333)^4 = 1.202 \text{ bar}$$

The fact that $p_5 > p_a = 0.54$ indicates that the nozzle is choked.

The critical pressure ratio of the nozzle is

$$p_{o4}/p_5 = [(k_g + 1)/2]^{kg/(kg-1)} = (2.333/2)^4 = 1.852$$

and the applied pressure ratio is 2.226/0.54 = 4.12. Thus the applied pressure ratio exceeds the critical pressure ratio.This also indicates that the nozzle is choked.

For the isentropic nozzle, the steady-flow energy equation gives

$$0 = h_5 + V_5^2/2 - h_{o4}$$

or, with $c_{p,g}$ constant,

$$V_5 = [2c_{p,g}(T_{o4} - T_5)]^{½} \qquad \text{[ft/s | m/s]} \qquad (5.47)$$

Thus the jet velocity is determined from Equation (5.47), where T_5 is obtained from Equation (5.45).

The *thrust* of the engine is obtained by applying Newton's Second Law to a control volume, as shown in Figure 5.22. If the mass flow rate through the engine is m, the rates of momentum flow into and out of the control volume are mV_a and mV_5, respectively. The net force exerted by the exit pressure is $(p_5 - p_a)A_5$, where A_5 is the nozzle exit area. Thus, applying Newton's Second Law to the control volume, we can relate the force exerted by the engine on the gases flowing through, F, and the net exit pressure force to the rate of increase of flow momentum produced by the engine:

$$m(V_5 - V_a) = F - (p_5 - p_a)A_5 \qquad [\text{lb}_f \text{ | kN}]$$

or

$$F = m(V_5 - V_a) + (p_5 - p_a)A_5 \qquad [\text{lb}_f \text{ | kN}] \qquad (5.48)$$

Here, F is the engine force acting on the gas throughflow. The reaction to this force is the *thrust* on the engine acting in the direction of flight. Thus the magnitude of the

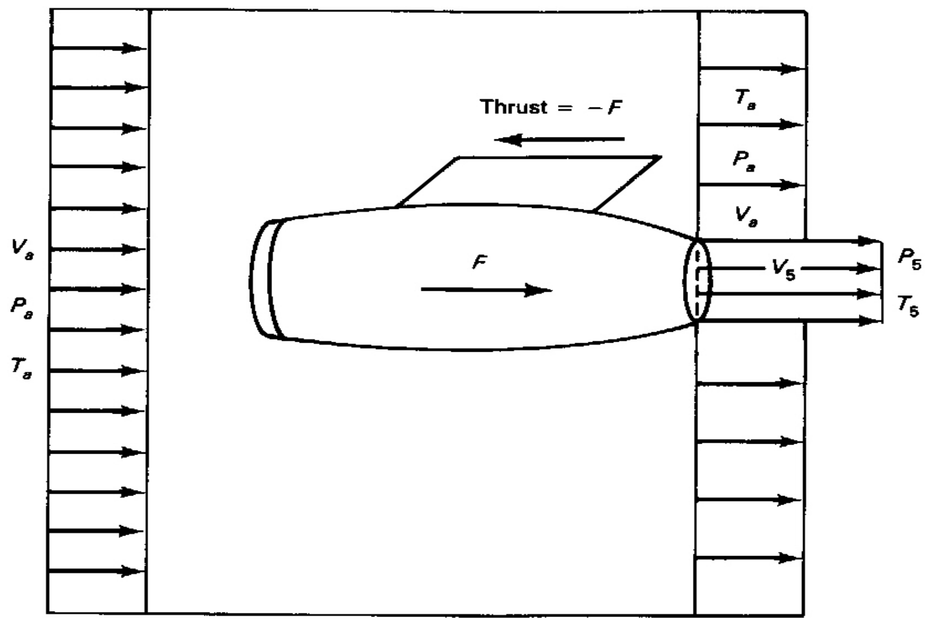

FIGURE 5.22 Control volume for the determination of engine thrust.

thrust is given by Equation (5.48). It is the sum of all the pressure force components acting on the inside the engine in the direction of flight.

The exit area, A_5, is related to the mass flow rate by

$$m = A_5 \rho_5 V_5 \qquad [\mathrm{lb_m/s} \mid \mathrm{kg/s}] \qquad (5.49)$$

where the density at station 5 is obtained from the perfect gas law using p_5 and T_5 from Equations (5.45) and (5.46). If A_5 is known, the mass flow rate through the engine may be determined from Equation (5.49) and the thrust from Equation (5.48).

Another type of nozzle used in high-performance engines and in rocket nozzles is a *convergent-divergent nozzle*, one in which the flow area first contracts and then increases. It differs from the convergent nozzle in that it can have supersonic flow at the exit. For such a fully expanded, convergent-divergent nozzle operating at its *design condition*, $p_5 = p_a$, and the engine thrust from Equation (5.48) reduces to $m(V_5 - V_a)$.

Jet Engine Performance

It is seen that engine thrust is proportional to the mass flow rate through the engine and to the excess of the jet velocity over the flight velocity. The specific thrust of an engine is defined as the ratio of the engine thrust to its mass flow rate. From Equation (5.48) the specific thrust is

$$F/m = (V_5 - V_a) + (p_5 - p_a)A_5/m \qquad [\text{lb}_f\text{-s/lb}_m \mid \text{kN-s/kg}] \qquad (5.50)$$

Because the engine mass flow rate is proportional to its exit area, as seen in Equation (5.49), A_5/m depends only on design nozzle exit conditions. As a consequence, F/m is independent of mass flow rate and depends only on flight velocity and altitude. Assigning an engine *design thrust* then determines the required engine-mass flow rate and nozzle exit area and thus the engine diameter. Thus the *specific thrust*, F/m, is an important engine design parameter for scaling engine size with required thrust at given flight conditions.

Another important engine design parameter is the *thrust specific fuel consumption,* TSFC, the ratio of the mass rate of fuel consumption to the engine thrust

$$\text{TSFC} = m_f/F \qquad [\text{lb}_m / \text{lb}_f\text{-s} \mid \text{kg / kN-s}] \qquad (5.51)$$

Low values of TSFC, of course, are favorable. The distance an aircraft can fly without refueling, called its *range*, is inversely proportional to the TSFC of its engines. The following example demonstrates the evaluation of these parameters.

EXAMPLE 5.6

An aircraft flies at a speed of 250 m/s at an altitude of 5000 m. The engines operate at a compressor pressure ratio of 8, with a turbine inlet temperature of 1200K. The compressor and turbine efficiencies are 0.9 and 0.87, respectively, and there is a 4% pressure loss in the combustion chamber. The inlet total pressure recovery is 0.97, and the engine-mass flow rate is 100 kg/s. Use an engine mechanical efficiency of 0.99 and a fuel heating value of 43,000 kJ/kg. Assume that the engine has a convergent, isentropic, nozzle flow. Determine the nozzle exit area, the engine thrust, specific thrust, fuel flow rate, and thrust specific fuel consumption.

Solution

The solution details are presented in Table 5.5 in spreadsheet form. At 5000 m altitude, the ambient static temperature and pressure are determined from standard-atmosphere tables such as those given in Appendix H. The ambient stagnation pressure and temperature are then determined for the given flight speed. The stagnation temperature at the compressor entrance is the same as the free-stream value for an adiabatic inlet. The inlet pressure recovery determines the total pressure at the compressor entrance. Using the notation of Figures 5.21 and 5.22, and given the compressor pressure ratio and turbine inlet temperature, the stagnation conditions at the compressor, combustor, and turbine exits may be determined in the same way as the static conditions were determined earlier for stationary two-shaft gas turbines.

TABLE 5.5 Spreadsheet Solution to Example 5.6

Turbojet analysis in SI units

Va	250	m/s	Flight velocity
Altitude	5000	m	given
Ta	255.7	K	Ambient temperature
Pa	0.5405	Bar	Ambient pressure
Fuel HV	43100	kJ/kg	Heating value
Cpa	1.005	kJ/kg	Air heat capacity
Cpa/Cpg	0.8754		Air-gas heat capacity ratio
r	8		Compressor pressure ratio
m	100	kg/s	Inlet mass flow rate
PR	0.97		Inlet pressure recovery
comp eta	0.9		Compressor efficiency
(k-1)/k	0.2857		Compressor isentropic factor for k = 1.4
turb eta	0.87		Turbine isentropic efficiecy
(kg-1)/kg	0.25		Turbine isentropic factor for k = 4/3
eta mech	0.99		mechanical efficiency
comb dp/p	0.04		Combustor fractional pressure loss
To3=	927	C	Turbine inlet temperature, C
To3=	1200	K	Turbine inlet temperature, K
To1=Toa=	286.79	K	Toa=Ta+Va^2/(2*Cpa*1000)
Poa=	0.808	Bar	Poa=Pa*(Toa/Ta)^[k/(k-1)]
Po1=	0.783	Bar	Po1=Poa*inlet pressure recovery
Po2=	6.268	Bar	Po2=r*Po1
Po3=	6.017	Bar	Po3=Po2*[1 - (comb dp/p)]
To2s=	519.50	K	To2s=To1*r^(k-1)/k
To2=	546.65	K	To2=To1+(To2s-T01)/(comp eta)
To4=	970.22	K	To4=To3-(Cpa/Cpg)*(To2-To1)/eta mech
To4s	935.88	K	To4s=To3-(To3-To4)/[turb eta]
Po4=	2.23	Bar	Po4=Po3*(To4/To3)^[kg/(kg-1)]
Po4/Pa	4.118		Nozzle flow is choked
Po4/Pc	1.85		Po4/Pc=[(k+1)/2]^[k/(k-1)]
T5=	831.62	K	IFPc>Pa,To4*2/(4/3+1),To4/(Po4/Pa)^(1/4)
V5=	564.11	m/s	IFPc>Pa,SQRT(287*T5*4/3),SQRT(2000(To4-T5)Cpg)
P5=	1.20	Bar	IFPo4/Pa>Po4/Pc,Po4/(Po4/Pc),Pa
rho5=	0.50	kg/m^3	rho5=100*P5/(.287*T5)
A5/m5=	0.0035	m^2-s/kg	A5/m5=1/(C5*rho5)
Spec.Thr	0.547	kN-s/kg	[(V5-Va)+(P5-Pa)*(A5/m5)*10^5)]/1000
thrust	54.70	kN	Total engine thrust
f/a=	0.0174		f/a=Cpg(To3-To2)/HV
mf	1.740	kg/s	Fuel flow rate=m*(f/a)
TSFC=	0.032	kg/kN-s	TSFC=(f/a)/specific thrust
TSFC=	114.5	kg/kN-hr	TSFC=3600*(f/a)/specific thrust

The calculated available nozzle pressure ratio $p_{04}/p_a = 4.118$ is then compared with the critical pressure ratio $p_{04}/p_c = 1.852$, which indicates that the convergent nozzle is choked; i.e., sonic velocity exists at the throat. The nozzle exit plane pressure must then be given by p_{04} divided by the critical pressure ratio. Equation (5.45) for a sonic condition then determines the nozzle exit plane temperature, and the exit plane density follows from the ideal gas law. Because the exit is choked, the exit plane temperature determines the exit velocity through the sonic velocity relation.

The nozzle exit area may then be determined by using the given mass flow rate and the exit velocity and density from Equation (5.49). The fuel-air ratio for the combustor is estimated from a simple application of the steady-flow energy equation to the combustor, which neglects the fuel sensible heat with respect to its heating value and assumes hot-air properties for the combustion products. The thrust, specific thrust, and TSFC are then found from Equations (5.48), (5.50), and (5.51).

The spreadsheet in Table 5.5 (available from Spreadsheet Examples as Example 5.6) is set up with conditional statements that treat the convergent nozzle for both the choked and subsonic exit conditions. The calculations of T_5, V_5, and p_5 depend on whether the throat is choked or not. The format for the spreadsheet conditional statements is:

Conditional test, Result if true, Result if not true

The low value of fuel-air ratio, $f = 0.0174$, obtained in this example is typical of most gas turbines and jet engines. In comparison with the stoichiometric value of 0.068, it corresponds to an equivalence ratio of 0.256.

Modern Jet Engines

Full and cutaway views of a small modern jet engine used in business aircraft are shown in Figure 5.23. The engine is a *turbofan engine*, a type of gas turbine engine that is used in all large commercial aircraft and is gaining popularity in the business jet market. The fan referred to in the turbofan name is seen at the left in Figure 5.23(b). The incoming air splits after passing through the fan, with the flow through the outer annulus passing to a nozzle that expels it without heating. The inner-core flow leaving the fan passes through an axial flow compressor stage and a centrifugal compressor before entering the combustor, turbine stages, and exit nozzle. Thus the core flow provides the turbine power to drive the fan and the compressors. The thrust specific fuel consumption and the thrust/weight ratio of turbofans is superior to those of conventional jet engines, because a larger mass flow rate of air is processed and exits at high velocity. The lower average exit velocity of the turbofan engine (compared to turbojets) is secondary in importance to the increased total mass flow rate through the engine.

A large turbofan engine designed to power Boeing 747 and 767, Airbus A310 and A300, and McDonnell-Douglas MD-11 aircraft is shown in Figure 5.24. In large turbofans and most large jet engines, axial compressors and turbines are used rather than centrifugal compressors. The axial compressors are capable of much higher pressure ratios, allow compression without turning the flow through a large angle, and have somewhat higher efficiencies than centrifugals. Large axial compressors have many axial stages and are capable of overall pressure ratios in excess of 30. Turbofans are studied in more detail in Chapter 9.

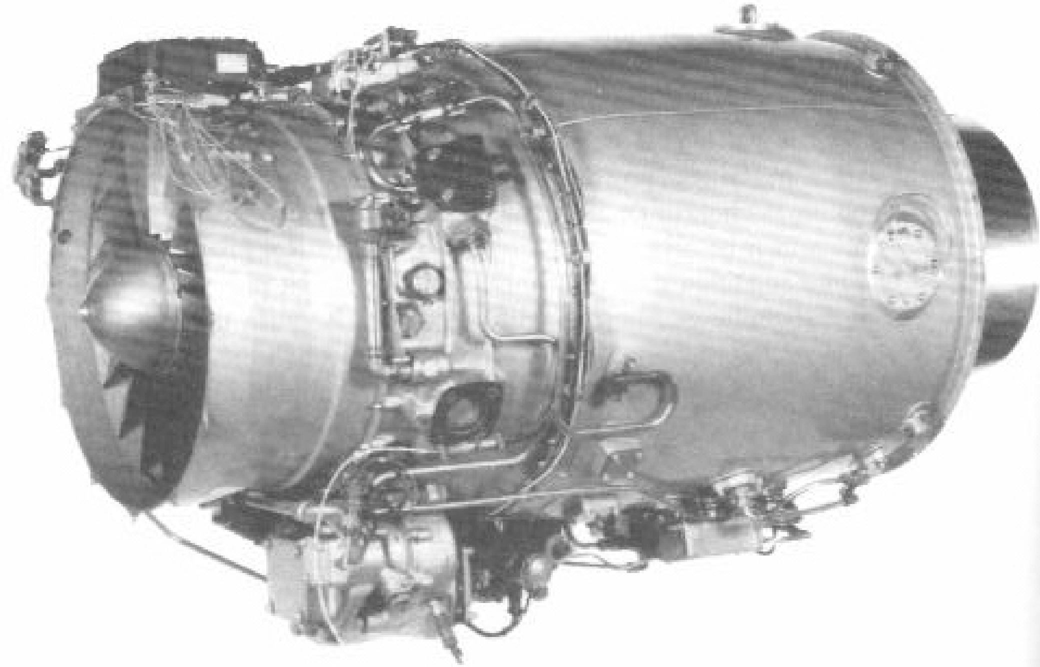

Figure 5.23a A small turbofan engine, the JT-15D. (Courtesy of Pratt and Whitney Canada.)

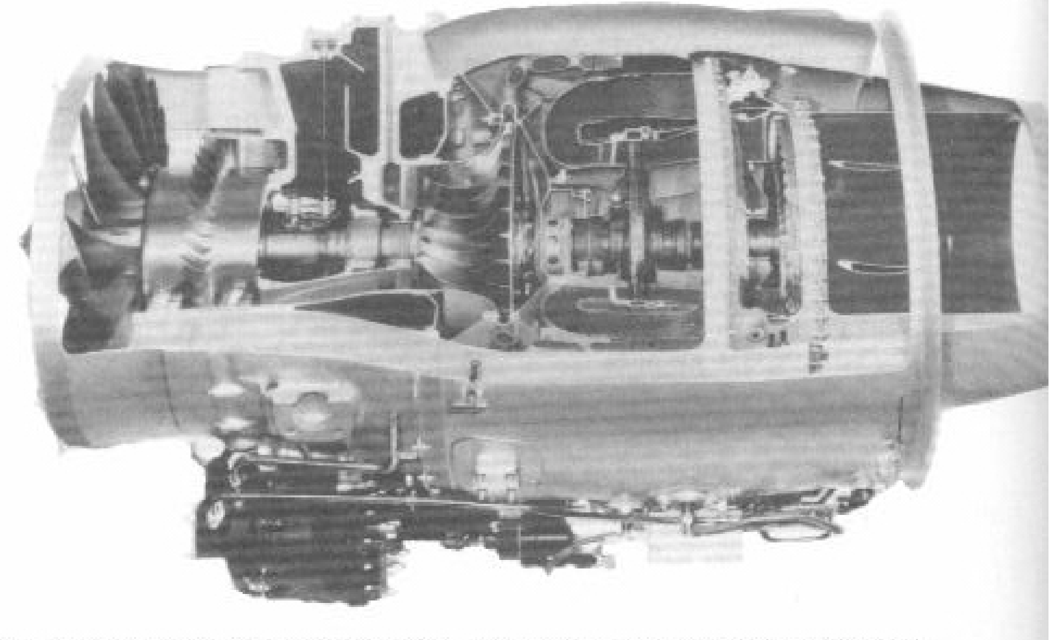

Figure 5.23b Cutaway of the JT-15D engine. (Courtesy of Pratt and Whitney Canada.)

Afterburning

The exhaust of a jet engine contains a large amount of unused oxygen because of the high air-fuel ratios necessary to limit the gas stagnation temperature to which the

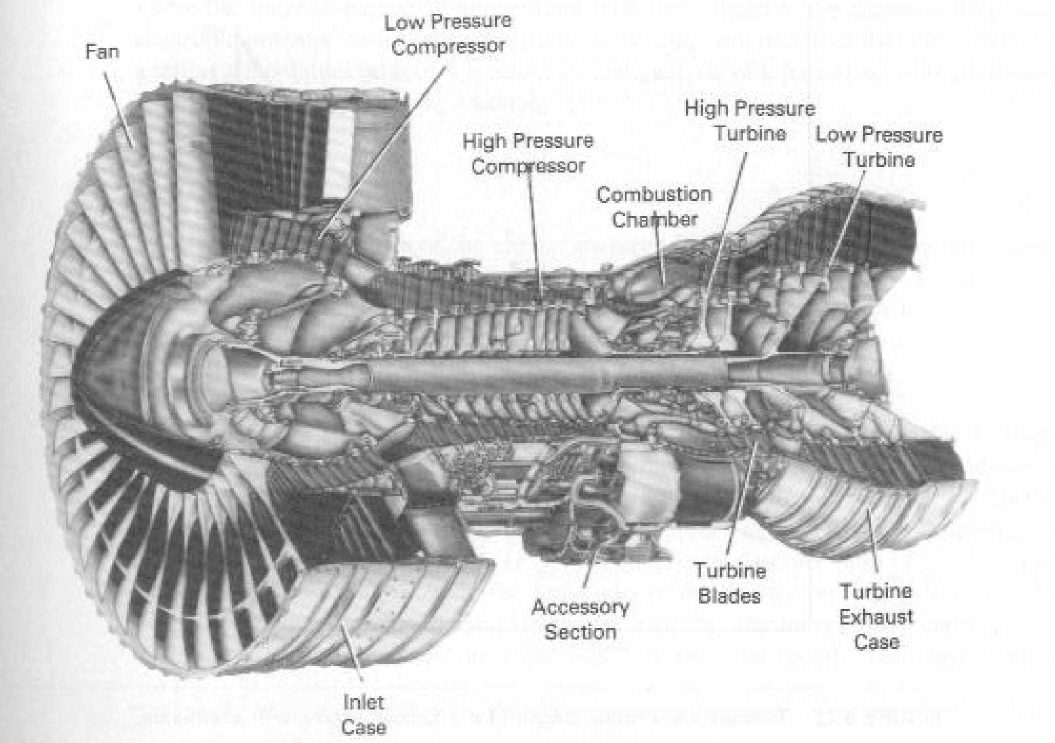

FIGURE 5.24 Large turbofan engine, the PW-4000. (Courtesy of Pratt and Whitney.)

turbine blading is exposed. This excess oxygen at the turbine exit makes it possible to burn additional fuel downstream and thereby to increase the nozzle exit temperature and jet velocity. By extending the interface between the turbine and the nozzle in a jet engine and by adding fuel spray bars to create a large combustion chamber called an *afterburner*, it is possible to dramatically increase the thrust of a jet engine. Much higher afterburner stagnation temperatures are allowed than those leaving the combustor, because (a) there is no highly stressed rotating machinery downstream of the afterburner, and (b) afterburner operating periods are usually limited to durations of a few minutes. Afterburners are used for thrust augmentation of jet aircraft to assist in takeoff and climb and to provide a brief high-speed-dash capability and increased maneuver thrust in military aircraft. However, the substantial fuel consumption penalty of afterburning restricts its use to brief periods of time when it is badly needed.

The T-s diagram for a jet engine with afterburner seen in Figure 5.25 shows that afterburning is analogous to reheating in an open-cycle stationary gas turbine, as shown in Figure 5.16. The energy release in the afterburner at approximately constant stagnation pressure shifts the nozzle expansion process to the right on the T-s diagram. As a result, the nozzle enthalpy and temperature differences between T-s diagram

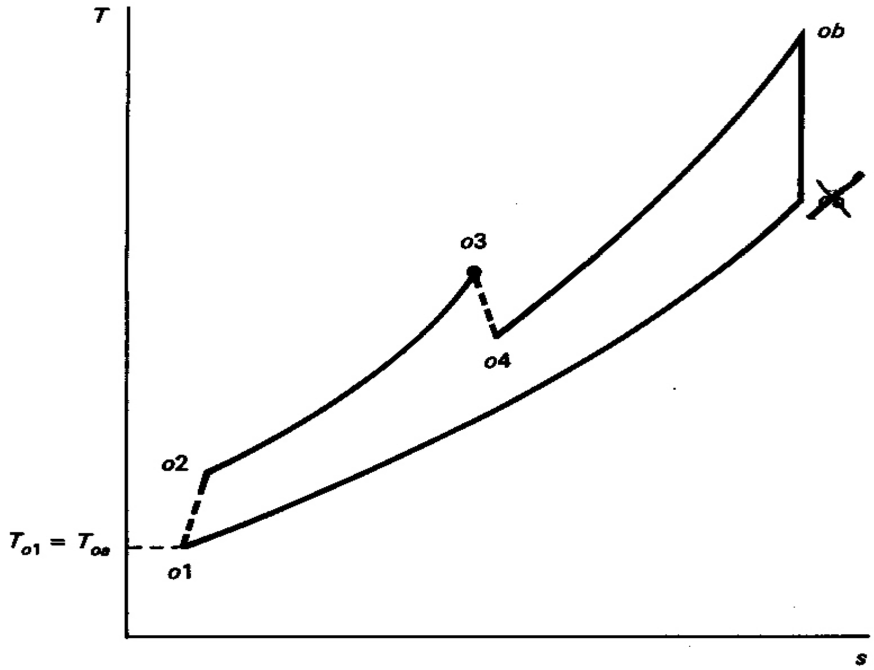

FIGURE 5.25 Temperature–entropy diagram for a turbojet engine with afterburner.

constant-pressure lines increase as the nozzle inlet stagnation temperature increases. This produces higher jet velocities. Using Equation (5.47) and the notation of Figure 5.16, we get for the ratio of fully expanded nozzle isentropic jet velocity with afterburning to that without:

$$V/V_{wo} = [(T_{om*} - T_4)/(T_{om} - T_{4*})]^{1/2}$$

$$= (T_{om*}/T_{om})^{1/2}\,[(1 - T_4/T_{om*})/(1 - T_{4*}/T_{om})]^{1/2}$$

$$= (T_{om*}/T_{om})^{1/2}$$

where the static-to-stagnation temperature ratios are eliminated using the corresponding equal isentropic pressure ratios. Thus, for example, the jet velocity ratio, and therefore the thrust ratio, for a reheat temperature ratio of 4 is about 2. The analysis of a jet engine with afterburner is illustrated in the following example.

EXAMPLE 5.7

Consider the performance of the engine analyzed in Example 5.6 when an afterburner is added. Assume that heat addition in the afterburner raises the nozzle entrance stagnation temperature to 2000 K, with a 5% stagnation pressure loss in the afterburner. What is the increase in nozzle exit temperature, jet velocity, and thrust?

TABLE 5.6 Spreadsheet Solution to Example 5.7

Turbojet analysis with afterburner in SI units

Va	250	m/s	Flight velocity
Altitude	5000	m	
Ta	255.7	K	Ambient temperature
Pa	0.5405	Bar	Ambient pressure
Fuel HV	43100	kJ/kg	Heating value
Cpa	1.005	kJ/kg	Air heat capacity
Cpa/Cpg	0.8754		Air-gas heat capacity ratio
r	8		Compressor pressure ratio
m	100	kg/s	Inlet mass flow rate
PR	0.97		Inlet pressure recovery
comp eta	0.9		Compressor efficiency
(k-1)/k	0.2857		Compressor isentropic factor
turb eta	0.87		Turbine isentropic efficiency
(kg-1)/kg	0.25		Turbine isentropic factor
eta mech	0.99		mechanical efficiency
comb dp/p	0.04		Combustor fractional pressure loss
To3=	927	C	Turbine inlet temperature, C
To3=	1200	K	Turbine inlet temperature, K
To1=Toa=	286.795	K	Toa=Ta+Va^2/(2*Cpa*1000)
Poa=	0.808	Bar	Poa=Pa*(Toa/Ta)^[k/(k-1)]
Po1=	0.783	Bar	Po1=Poa*inlet pressure recovery
Po2=	6.268	Bar	Po2=r*Po1
Po3=	6.017	Bar	Po3=Po2*[1 - (comb dp/p)]
To2s=	519.50	K	To2s=To1*r^(k-1)/k
To2=	546.65	K	To2=To1+(To2s-T01)/(comp eta)
To4=	970.22	K	To4=To3-(Cpa/Cpg)*(To2-To1)/eta mech
To4s	935.88	K	To4s=To3-(To3-To4)/[turb eta]
Po4=	2.226	Bar	Po4=Po3*(To4s/To3)^[kg/(kg-1)]
delp	0.05		fractional pressure loss - given
Tob	2000	K	given afterburner temperature
Pob	2.115	Bar	Pob=Po4(1-delp)
Pob/Pa	3.913		Nozzle flow is choked
Pob/Pc	1.852		Pob/Pc=[(k+1)/2]^[k/(k-1)]
T5=	1714.29	K	IFP5>Pa,To4*2/(4/3+1),To4/(Po4/Pa)^(1/4)
V5=	809.93	m/s	IFP5>Pa,SQRT(287*T5*4/3),SQRT(2000(To4-T5)Cpg)
P5=	1.142	Bar	IFPob/Pa>Pob/Pc,Pob/(Pob/Pc),Pa
rho5=	0.232	kg/m^3	rho5=100*P5/(.287*T5)
A5/m5=	0.0053	m^2-s/kg	A5/m5=1/(C5*rho5)
Spec.Thr=	0.880	kN-s/kg	[(V5-Va)+(P5-Pa)*(A5/m5)*10^5)]/1000
thrust	87.99	kN	Total engine thrust
f/a=	0.0448		f/a=Cpg(To3-To2+Tob-To4)/HV
mf	4.483	kg/s	Fuel flow rate=m*(f/a)
TSFC=	0.051	kg/kN-s	TSFC=(f/a)/specific thrust
TSFC=	183.4	kg/kN-hr	TSFC=3600*(f/a)/specific thrust

Solution

Table 5.6 is from an adaptation of the spreadsheet (Table 5.5) used for Example 5.6. The calculations are identical up to the turbine exit at station 4. Heat addition in the afterburner raises the stagnation temperature at the nozzle entrance, $T_{ob,}$ to 2000K, while the 5% pressure loss drops the stagnation pressure to 2.115 bar. Comparing the applied nozzle pressure ratio (p_{ob}/p_a) with the critical pressure ratio (p_{ob}/p_c) shows that the nozzle remains choked. The remainder of the calculation follows Example 5.6 except that the fuel consumption associated with the afterburner temperature rise is taking into account in the fuel-air ratio, fuel flow rate, and specific fuel consumption.

Table 5.7 compares some of the performance parameters calculated in Examples 5.6 and 5.7. The table shows clearly the striking gain in thrust provided by the high nozzle exit temperature produced by afterburning and the accompanying high penalty in fuel consumption. Note, however, that even with afterburning the overall equivalence ratio 0.0448/0.068 = 0.659 is well below stoichiometric.

Table 5.7 Examples 5.6 and 5.7 Compared

Parameter	Without Afterburner	With Afterburner
Temperature, K		
Nozzle entrance	970.2	2000.0
Nozzle exit	831.6	1714.3
Nozzle exit velocity, m/s	564.1	809.9
Thrust, kN	54.7	87.99
Fuel-air Ratio	0.0174	0.0448
TSFC, kg/kN-s	0.032	0.051

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EXERCISES

5.1 For the Air Standard Brayton cycle, express the net work in terms of the compressor pressure ratio, r, and the turbine-to-compressor inlet temperature ratio, T_3/T_1. Nondimensionalize the net work with c_pT_1, and derive an expression for the pressure ratio that maximizes the net work for a given value of T_3/T_1.

5.2 For a Brayton Air Standard cycle, work out an expression for the maximum possible compressor pressure ratio for a given turbine-to-compressor inlet temperature ratio. Draw and label the cycle on a T-s diagram. What is the magnitude of the net work for this cycle? Explain.

5.3 For a calorically perfect gas, write an expression for the temperature difference, $T_2 - T_1$, on an isentrope between two lines of constant pressure in terms of the initial temperature T_1 and the pressure ratio p_2/p_1. Sketch a T-s diagram showing two different isentropes between the two pressure levels. Explain how your expression demonstrates that the work of an isentropic turbomachine operating between given pressure levels increases with temperature.

5.4 Derive an expression for the enthalpy difference, $h_2 - h_1$, along a calorically perfect gas isentrope spanning two fixed pressure levels, p_2 and p_1, in terms of the discharge temperature T_2. Note that as T_2 increases, the enthalpy difference also increases.

5.5 Derive Equation (5.9).

5.6 Derive Equations (5.24) and (5.25).

5.7 A simple-cycle stationary gas turbine has compressor and turbine efficiencies of 0.85 and 0.9, respectively, and a compressor pressure ratio of 20. Determine the work of the compressor and the turbine, the net work, the turbine exit temperature, and the thermal efficiency for 80°F ambient and 1900°F turbine inlet temperatures.

5.8 A simple-cycle stationary gas turbine has compressor and turbine efficiencies of 0.85 and 0.9, respectively, and a compressor pressure ratio of 20. Determine the work of the compressor and the turbine, the net work, the turbine exit temperature, and the thermal efficiency for 20°C ambient and 1200°C turbine inlet temperatures.

5.9 A regenerative-cycle stationary gas turbine has compressor and turbine isentropic efficiencies of 0.85 and 0.9, respectively, a regenerator effectiveness of 0.8, and a compressor pressure ratio of 5. Determine the work of the compressor and the turbine, the net work, the turbine and regenerator exit temperatures, and the thermal efficiency for 80°F ambient and 1900°F turbine inlet temperatures. Compare the efficiency of the cycle with the corresponding simple-cycle efficiency.

5.10 A regenerative-cycle stationary gas turbine has compressor and turbine isentropic efficiencies of 0.85 and 0.9, respectively, a regenerator effectiveness of 0.8, and a compressor pressure ratio of 5. Determine the work of the compressor and the turbine, the net work, the turbine and regenerator exit temperatures, and the thermal efficiency for 20°C ambient and 1200°C turbine inlet temperatures. Compare the efficiency of the cycle with the corresponding simple-cycle efficiency.

5.11 A two-shaft stationary gas turbine has isentropic efficiencies of 0.85, 0.88, and 0.9

for the compressor, gas generator turbine, and power turbine, respectively, and a compressor pressure ratio of 20.

(a) Determine the compressor work and net work, the gas generator turbine exit temperature, and the thermal efficiency for 80°F ambient and 1900°F compressor-turbine inlet temperatures.

(b) Calculate and discuss the effects of adding reheat to 1900°F ahead of the power turbine.

5.12 A two-shaft stationary gas turbine has isentropic efficiencies of 0.85, 0.88, and 0.9 for the compressor, gas generator turbine, and power turbine, respectively, and a compressor pressure ratio of 20.

(a) Determine the compressor work and net work, the gas generator turbine exit temperature, and the thermal efficiency for 20°C ambient and 1200°C compressor-turbine inlet temperatures.

(b) Calculate and discuss the effects of adding reheat to 1200°C ahead of the power turbine.

5.13 A two-shaft stationary gas turbine with an intercooler and reheater has efficiencies of 0.85, 0.88, and 0.9 for the compressor, gas-generator turbine, and power turbine, respectively, and a compressor pressure ratio of 5.

(a) Determine the compressor work and net work, the gas generator turbine exit temperature, and the thermal efficiency for 80°F ambient and 1900°F turbine inlet temperatures.

(b) Calculate and discuss the effect on thermal efficiency, exhaust temperature, and net work of adding a regenerator with an effectiveness of 75%.

5.14 A two-shaft stationary gas turbine with an intercooler and reheater has efficiencies of 0.85, 0.88, and 0.9 for the compressor, gas-generator turbine, and power turbine, respectively, and a compressor pressure ratio of 5.

(a) Determine the compressor work and net work, the gas generator turbine exit temperature, and the thermal efficiency for 20°C ambient and 1200°C turbine inlet temperatures.

(b) Calculate and discuss the effect on thermal efficiency, exhaust temperature, and net work of adding a regenerator with an effectiveness of 75%.

5.15 Consider a pulverized-coal-burning, single-shaft gas turbine in which the combustion chamber is downstream of the turbine to avoid turbine blade erosion and corrosion. The combustion gases leaving the burner heat the compressor discharge air through the intervening walls of a high temperature ceramic heat exchanger.

(a) Sketch the flow and T-s diagrams for this gas turbine, showing the influence of pressure drops through the combustor and the heat exchanger. The ambient, turbine inlet, and combustor exhaust temperatures are 80°F, 1900°F, and 3000°F, respectively. The compressor pressure ratio is 5. Assume perfect turbomachinery.

(b) For zero pressure drops, determine the net work, the thermal efficiency, and the heat exchanger exhaust temperature.

(c) If the coal has a heating value of 14,000 Btu/lb_m, what is the coal consumption rate, in tons per hour, for a 50-MW plant?

5.16 Consider a pulverized-coal-burning, single-shaft gas turbine in which the combustion chamber is downstream of the turbine to avoid turbine blade erosion and corrosion. The combustion gases leaving the burner heat the compressor discharge air through the intervening walls of a high-temperature ceramic heat exchanger.

(a) Sketch the flow and T-s diagrams for this gas turbine, showing the influence of pressure drops through the combustor and the heat exchanger. The ambient, turbine inlet, and combustor exhaust temperatures are 20°C, 1200°C, and 2000°C, respectively. The compressor pressure ratio is 5. Assume perfect turbomachinery.

(b) For zero pressure drops, determine the net work, the thermal efficiency, and the heat exchanger exhaust temperature.

(c) If the coal has a heating value of 25,000 kJ/kg, what is the coal consumption rate, in tons per hour, for a 50-MW plant?

5.17 A stationary gas turbine used to supply compressed air to a factory operates with zero external shaft load. Derive an equation for the fraction of the compressor inlet air that can be extracted ahead of the combustion chamber for process use in terms of the compressor pressure ratio, the ratio of turbine-to-compressor inlet temperatures, and the turbomachinery efficiencies. Plot the compressor mass extraction ratio as a function of compressor pressure ratio for temperature ratios of 3 and 5, perfect turbomachinery, and identical high- and low-temperature heat capacities.

5.18 A stationary gas turbine used to supply compressed air to a factory operates with zero external shaft load. Derive an equation for the fraction of the inlet air that can be extracted ahead of the combustion chamber for process use in terms of the compressor pressure ratio, the ratio of turbine-to-compressor inlet temperatures, and the turbomachinery efficiencies. What is the extraction mass flow for a machine that has a compressor pressure ratio of 10, turbomachine inlet temperatures of 1800°F and 80°F, turbomachine efficiencies of 90%, and a compressor inlet mass flow rate of 15 lb_m/s?

5.19 A stationary gas turbine used to supply compressed air to a factory operates with zero external shaft load. Derive an equation for the fraction of the inlet air that can be extracted ahead of the combustion chamber for process use in terms of the compressor pressure ratio, the ratio of turbine-to-compressor inlet temperatures, and the turbomachinery efficiencies. What is the extraction mass flow for a machine that has a compressor pressure ratio of 8, turbomachine inlet temperatures of 1000°C and 25°C, turbomachine efficiencies of 90%, and a compressor inlet mass flow rate of 10 kg/s?

5.20 Do Exercise 5.18, but account for combustion chamber fractional pressure drops of 3% and 5% of the burner inlet pressure. How does increased pressure loss influence process mass flow?

5.21 Do Exercise 5.19, but account for combustion chamber fractional pressure drops of 3% and 5% of the burner inlet pressure. How does increased pressure loss influence process mass flow?

5.22 A gas-turbine-driven car requires a maximum of 240 shaft horsepower. The engine is a two-shaft regenerative gas turbine with compressor, gas generator turbine, and power turbine efficiencies of 0.86, 0.9, and 0.87, respectively, and a regenerator effectiveness of 0.72. The compressor pressure ratio is 3.7, and the turbine and compressor inlet temperatures are 1800°F and 90°F, respectively. What air flow rate does the engine require? What is the automobile exhaust temperature? What are the engine fuel-air ratio and specific fuel consumption if the engine burns gaseous methane?

5.23 A gas-turbine-driven car requires a maximum of 150kW of shaft power. The engine is a two-shaft regenerative gas turbine with compressor, gas generator turbine, and power turbine efficiencies of 0.84, 0.87 and 0.9, respectively, and a regenerator effectiveness of 0.75. The compressor pressure ratio is 4.3, and the turbine and compressor inlet temperatures are 1250°C and 20°C, respectively. What air flow rate does the engine require? What is the automobile exhaust temperature? What are the engine fuel-air ratio and specific fuel consumption if the engine burns gaseous hydrogen?

5.24 A simple-cycle gas turbine is designed for a turbine inlet temperature of 1450°F, a compressor pressure ratio of 12, and compressor and turbine efficiencies of 84% and 88%, respectively. Ambient conditions are 85°F and 14.5 psia.

(a) Draw and label a T-s diagram for this engine.

(b) Determine the compressor, turbine, and net work for this cycle.

(c) Determine the engine thermal efficiency.

(d) Your supervisor has requested that you study the influence of replacing the compressor with dual compressors and an intercooler. Assume that the new compressors are identical to each other and have the same efficiencies and combined overall pressure ratio as the original compressor. Assume intercooling to 85°F with no intercooler pressure losses. Show clearly the T-s diagram for the modified system superimposed on your original diagram. Calculate the revised system net work and thermal efficiency.

5.25 Determine the air and kerosene flow rates for a 100-MW regenerative gas turbine with 1800K turbine inlet temperature, compressor pressure ratio of 5, and 1 atm. and 300K ambient conditions. The compressor and turbine efficiencies are 81% and 88%, respectively, and the heat exchanger effectiveness is 75%. Use a heating value for kerosene of 45,840 kJ/kg. What is the engine specific fuel consumption?

5.26 A regenerative gas turbine has compressor and turbine discharge temperatures of 350K and 700K, respectively. Draw and label a T-s diagram showing the relevant states. If the regenerator has an effectiveness of 70%, what are the combustor inlet temperature and engine exhaust temperature?

5.27 A gas turbine has a turbine inlet temperature of 1100K, a turbine pressure ratio of 6, and a turbine efficiency of 90%. What are the turbine exit temperature and the turbine work?

5.28 A two-shaft gas turbine with reheat has turbine inlet temperatures of 1500°F, a compressor pressure ratio of 16, and turbomachine efficiencies of 88% each. The compressor inlet conditions are 80°F and 1 atm. Assume that all heat capacities are 0.24 Btu/lb_m-R and $k = 1.4$.

(a) Draw T-s and flow diagrams.
(b) Make a table of temperatures and pressures for all real states, in °R and atm.
(c) What are the compressor work and power turbine work?
(d) What is the power-turbine-to-compressor work ratio?
(e) What is the cycle thermal efficiency?
(f) Evaluate the recommendation to add a regenerator to the system. If a 4-count (0.04) increase in thermal efficiency can be achieved, the addition of the regenerator is considered economically feasible. Give your recommendation, supporting arguments, and substantiating quantitative data.

5.29 Consider a gas turbine with compressor and turbine inlet temperatures of 80°F and 1200°F, respectively. The turbine efficiency is 85%, and compressor pressure ratio is 8.

(a) Draw coordinated T-s and plant diagrams.
(b) What is the turbine work?
(c) What is the minimum compressor efficiency required for the gas turbine to produce a net power output?
(d) What is the thermal efficiency if the compressor efficiency is raised to 85%?

5.30 The first closed-cycle gas turbine power plant in the world using helium as a working fluid is a 50-MW plant located in Oberhausen, Germany (ref. 8). It was designed as an operating power plant and as a research facility to study aspects of component design and performance with helium as a working fluid. It has two compressors, with intercooling, connected directly to a high-pressure turbine. The high-pressure turbine is in turn connected through a gearbox to a low-pressure turbine with no reheat. Helium is heated first by regenerator, followed by a specially designed heater that burns coke-oven gas. A water-cooled pre-cooler returns the helium to the low-pressure compressor inlet conditions. The high-pressure turbine mass flow rate is 84.4 kg/s, and the heater efficiency is 92.2%. The following design data are given in the reference:

	Temperature, °C	Pressure, Bar
1. Low-pressure compressor inlet	25	10.5
2. Intercooler inlet	83	15.5
3. High-pressure compressor inlet	25	15.4
4. Regenerator inlet, high-pressure side	125	28.7
5. Heater inlet	417	28.2
6. High-pressure turbine inlet	750	27.0
7. Low-pressure turbine inlet	580	16.5
8. Regenerator inlet, low-pressure side	460	10.8
9. Precooler inlet	169	10.6

In the following, assume $k = 1.67$ and $c_p = 5.197$ kJ/kg-K for helium.

(a) Sketch and label flow and T-s diagrams for the plant.

(b) What is the overall engine pressure ratio of the gas turbine?

(c) Estimate the mechanical power output and plant thermal efficiency. Reference 8 gives 50-MW and 31.3% as net electrical output and efficiency, respectively. Evaluate your calculations with these data.

5.31 Determine the thrust for a turbojet engine flying at 200 m/s with a compressor inlet temperature of 27°C, a compressor pressure ratio of 11, a turbine inlet temperature of 1400K, and compressor and turbine efficiencies of 0.85 and 0.9, respectively. The engine mass flow rate is 20 kg/s. You may use $k = 1.4$ and c_p = 1.005 kJ/kg-K throughout and neglect the differences between static and stagnation properties in the turbomachinery. Assume an ambient pressure of 0.2615 atmospheres.

5.32 The gas generator of a two-shaft gas turbine has a compressor pressure ratio of 5 and compressor and turbine inlet temperatures of 80°F and 2000°F, respectively, at sea level. All turbomachines have efficiencies of 90%, and the inlet air flow is 50 lb_m/s.

(a) What are the net work, pressure ratio, and horsepower of the power turbine and the cycle efficiency?

(b) Suppose the power turbine is removed and the gas generator exhaust gas flows isentropically through a convergent-divergent propulsion nozzle that is fully expanded (exit pressure is ambient). What are the nozzle exhaust velocity and the static thrust?

(c) Repeat part (b) for a choked conversion nozzle.

5.33 A gas turbine with reheat has two turbines with efficiencies e_1 and e_2. Derive relations for the turbine pressure ratios r_1 and r_2 that maximize the total turbine work for a given overall turbine pressure ratio, r, if both turbines have the same inlet temperature. How do the pressure ratios compare if the turbine efficiencies are equal?

If the efficiency of one turbine is 50% higher than the other, what is the optimum pressure ratio?

5.34* For a simple-cycle gas turbine, develop a multicolumn spreadsheet that tabulates and plots (a) the net work, nondimensionalized by using the product of the compressor constant-pressure heat capacity and the compressor inlet temperature, and (b) the thermal efficiency, both as a function of compressor pressure ratio (only one graph with two curves). Use 80°F and 2000°F as compressor and turbine inlet temperatures, respectively, and 0.85 and 0.90 as compressor and turbine efficiencies, respectively. Use appropriate constant heat capacities. Each of the input values should be entered in separate cells in each column so that the spreadsheet may be used for studies with other parametric values. Use your plot and table to determine the pressure ratio that yields the maximum net work. Compare with your theoretical expectation.

5.35* Solve Exercise 5.34 for a regenerative cycle. Use a nominal value of 0.8 for regenerator effectiveness.

5.36* Solve Exercise 5.34 for a two-shaft regenerative cycle. Use a nominal value of 0.8 for regenerator effectiveness and 0.88 for the power turbine efficiency. Account also for 3% pressure losses in both sides of the regenerator.

5.37 For the conditions of Exercise 5.31, but using more realistic properties in the engine hot sections, plot curves of thrust and specific fuel consumption as a function of gaseous hydrogen air-fuel ratio. Determine the maximum thrust corresponding to the stoichiometric limit for gaseous hydrogen fuel.

5.38* Use the spreadsheet corresponding to Table 5.4 to plot a graph showing the influence of turbine inlet temperature on net work and thermal efficiency for two-shaft gas turbines, with and without regeneration, for a compressor pressure ratio of 4.

5.39 Extend Example 5.6 by using hand calculations to evaluate the thrust and the thrust specific fuel consumption for the engine if it were fitted with a fully expanded (exit pressure equal to ambient pressure) convergent-divergent isentropic nozzle.

5.40* Modify the spreadsheet corresponding to Table 5.6 to evaluate the thrust and the thrust specific fuel consumption for the engine if it were fitted with a fully expanded (exit pressure equal to ambient pressure) convergent-divergent isentropic nozzle.

5.41 A gas turbine engine is being designed to provide work and a hot, high-velocity exhaust flow. The compressor will have a pressure ratio of 4 and an isentropic efficiency of 90% at the design point. The compressor and load are driven by separate

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* Exercise numbers with asterisks involve computer usage.

turbines, but the overall expansion pressure ratio across the turbines will be 3 to 1, and the efficiency of each turbine will be 90%. The exhaust of the low-pressure turbine is expanded through a convergent nozzle to provide the high-velocity exhaust. At the design point the turbine inflow air temperature will be held to 1140°F, and the air flow rate will be 300,000 pounds mass per hour. Ambient conditions are 60°F and 14.7 psia. Calculate the brake power of the engine, in kW, and the temperatures at the entrance and the exit of nozzle.

5.42 A ramjet is a jet engine that flies at speeds high enough that the pressure rise produced by ram effect in the inlet makes a compressor and turbine unnecessary. At 50,000 feet and Mach 3, the inlet has a stagnation pressure recovery of 85%. Combustion raises the air temperature to 2500K. What is the thrust per unit mass flow rate of air and the exit velocity of the engine if the nozzle is (a) convergent, and (b) fully expanded (exit pressure equal to ambient pressure) convergent-divergent?

5.43 Using the First Law of Thermodynamics, derive an equation for the work of compression in a reversible steady flow in terms of volume and pressure. Use the equation to derive an expression for the reversible isothermal work of compression of a calorically perfect gas, with compressor pressure ratio as an independent variable.

5.44 Sketch a pressure-volume diagram comparing isothermal and isentropic compressions starting at the same state and having the same pressure ratio. Show for a thermally perfect gas that, at a given state, the isentrope has a steeper (negative) slope than an isotherm. Use your diagram to prove that the isothermal work of compression is less than the isentropic work.

5.45 A supersonic aircraft flies at Mach 2 at an altitude of 40,000 feet. Its engines have a compressor pressure ratio of 20 and a turbine inlet temperature of 2000°F. The inlets have total pressure recoveries of 89%, the compressors and turbines all have efficiencies of 90%, and the nozzles are convergent and isentropic. Determine the nozzle exit velocity and thrust, and estimate the thrust specific fuel consumption of each engine. The mass flow rate of air of each engine is 750 lb_m/s. Use a fuel heating value of 18,533 Btu/lb_m.

5.46 A supersonic aircraft flies at Mach 2 at an altitude of 13,000 m. Its engines have a compressor pressure ratio of 20 and a turbine inlet temperature of 1500K. The inlets have total pressure recoveries of 89%, the compressors and turbines all have efficiencies of 90%, and the nozzles are convergent and isentropic. Determine the nozzle exit velocity and thrust, and estimate the thrust specific fuel consumption of an engine for an engine air mass flow rate of 100 kg/s. Use a fuel heating value of 43,100 kJ/kg.

5.47 A supersonic aircraft flies at Mach 2 at an altitude of 40,000 feet. Its engines have a compressor pressure ratio of 20 and a turbine inlet temperature of 2000°F. The inlets have total pressure recoveries of 89%, and the compressors and turbines all have efficiencies of 90%. The engines have afterburners that raise the temperature of the gas entering the nozzles to 3000°F. The nozzles are convergent and isentropic. Compare the design thrust and thrust specific fuel consumption with the afterburner on and off for an engine air mass flow rate of 100 lb_m/s.

5.48* For a simple-cycle gas turbine, develop a multicolumn spreadsheet that tabulates and plots (a) the net work, nondimensionalized by using the compressor constant-pressure heat capacity and the compressor inlet temperature, and (b) the thermal efficiency, as a function of compressor pressure ratio (only one graph with two curves). Use 30° C and 1500°C as compressor and turbine inlet temperatures, respectively, and 0.85 and 0.90 as compressor and turbine efficiencies, respectively. Assume appropriate constant heat capacities. Each of the input values should be entered in separate cells in each column, so that the spreadsheet may be used for studies with other parametric values. Use your plot and table to determine the pressure ratio that yields the maximum net work. Compare with your theoretical expectation.

5.49* Solve Exercise 5.48 for a regenerative cycle. Use a nominal value of 0.85 for regenerator effectiveness.

5.50 Methane is burned in an adiabatic gas turbine combustor. The fuel enters the combustor at the reference temperature for the JANAF tables and mixes with air compressed from 80°F through a pressure ratio of 16 with a compressor efficiency of 90.5%. Determine the equivalence ratio that limits the turbine inlet temperature to 2060°F by:

(a) Using the JANAF tables.

(b) Using an energy balance on the combustion chamber and the lower heating value for methane.