LECTURE 11

EET 101 CIRCUIT ANALYSIS I

LESSON NO. 11

LESSON TITLE: CAPACITORS

I. Transients in Capacitor Networks

A. There are two phases in capacitor networks: Charge, Discharge

B. Charge Phase

1. When charging a capacitor, we need to think in terms of the charge stored in Coulombs.

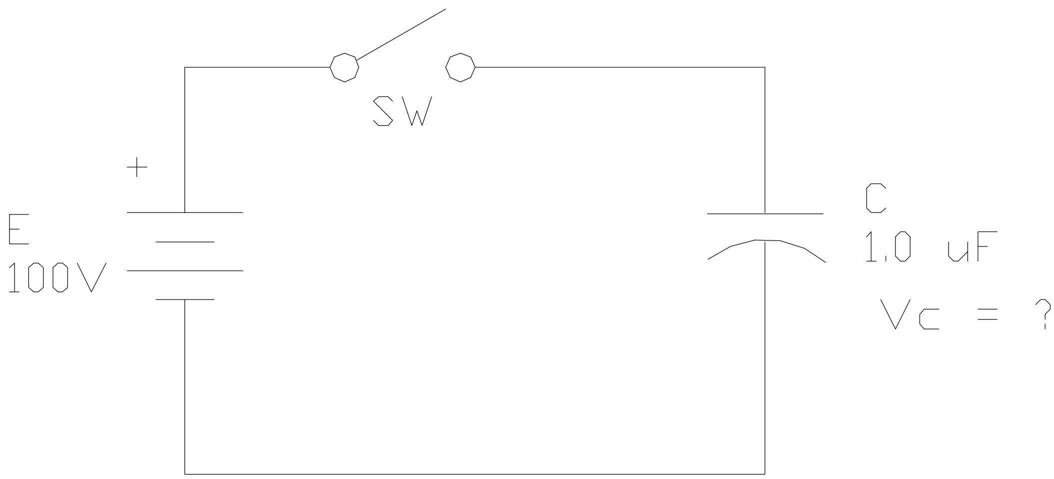
2. The charge phase is the transfer of electrons from the voltage source to the plates of the capacitor until the potential difference on the cap equals the applied voltage.

3. The current in the capacitor is the amount of charge that passes, or the change in charge placed on the plates, in an extremely small amount of time.

$i = d_Q/d_t$ $\qquad$ $Q = CV$

$\therefore i = C(dV_{CAP}/d_t)$

4. Circuit Schematic



a. Initial conditions, no charge on cap, switch open, no current flowing.

b. When S is open, the voltage on cap V_C = zero

c. Close the switch, the voltage will begin to rise on the cap as current (electrons) flow, depositing a charge on the lower plate. Electrons leave the upper plate and flow towards the source.

d. With no resistance in the circuit, the initial current will be extremely high.

e. Let R = 0.5 , the resistance of the wires.

f. Then I = E/R

g. The time will be a few microseconds.

h. Thus a short duration pulse of high current

i. A cap can be represented by a short-circuit at the beginning of a charge phase

j. A cap can be represented by an open-circuit when it is at the applied voltage potential

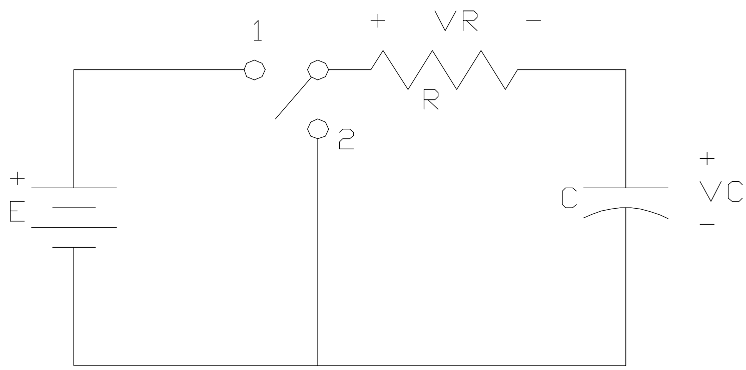
C. Kirchhoffs Voltage Law for Capacitors

1. Scripting notations:

   a. Steady-state conditions use UPPER case letters

   b. Instantaneous values of voltage, current and time are represented by lower case letters.

2. Circuit Schematic



2. $E = iR + V_{CAP}$

   $E = C(dV_{CAP}/d_t)R + V_{CAP}$

3. By the use of Calculus; $V_{CAP} = E(1\text{-}e^{-t/RC})$

4. RC = τ (tau) This is the time constant, in seconds.

   $\therefore$ $V_C = E(1\text{-}e^{-t/\tau})$

5. To charge a capacitor it take 5 time constants. Each time constant being equal to 63.2% of the remaining voltage for the cap to charge up to. Switch in position 1 in above schematic.

6. The capacitor will charge up to 63.2 % of final voltage in the 1^{st} time constant. See following table.

Time Constant	% of Final Voltage
1^{st}	63.2 %
2^{nd}	86.5 %
3^{rd}	95 %
4^{th}	98.2 %
5^{th}	99.3 %

D. Capacitor Current formula

1. $i_C = (E/R)e^{-t/\tau}$

E. Universal Charge/discharge curve, shows the charging and discharging curve for the capacitor in terms of voltage versus time constants.

F. Work Example 10.5

G. Discharge Phase

1. Discharging a capacitor is basically the same principle as that of charging a capacitor. Switch in position 2 in above schematic.

   a. When charging a capacitor, initial I = V_{INIT}/R

   b. When discharging a cap, $dv/dt = -V_{INIT}/RC$

2. The capacitor will discharge 63.2% in one time constant (τ), or drop to 36.8% of its initial voltage value. See following table.

Time Constant	% of Initial Voltage
1^{st}	36.8 %
2^{nd}	13.5 %
3^{rd}	5 %
4^{th}	1.8 %
5^{th}	0.67 %

3. The loop voltage must be equal across the resistor and the capacitor. $V_C = V_R$

   a. $i_C = E/R(e^{-t/RC})$ where E = V of Capacitor

   b. $V_C = Ee^{-t/RC}$ where E = V of Capacitor

H. Work Example 10.6

II. Instantaneous Values

A. The value of voltage to which a capacitor has charged up to at any one point in time.

B. The value at some portion of the charge/discharge curve that is not at the normal time constant intervals. Provides additional accuracy.

C. Therefore: $V_C = E(1-e^{-t/\tau})$ t = any time you desire, in seconds.

D. To find the time for a capacitor to charge to a certain voltage:

$$t = \tau \log_e(E/VC)$$

III. Thevenin Resistance for the Capacitor

A. A complex charging network must be Thevenized for the Capacitor to find the resistance for the time constant.

1. Open the Capacitor circuit

2. Short out power supply

3. Solve for the series-parallel equivalent resistance (R_{TH})

4. Put power supply back in the circuit

4. Use voltage divider techniques to find the voltage as seen by the capacitor (E_{TH})

B. Work Example 10.10

IV. Capacitor Current i_C

A. The current associated with a capacitance is related to the changing voltage across a capacitor.

B. $i_C = C(dV_C/dt)$

C. If the capacitor voltage change is zero, than the current is zero.

D. The average current is define as the change in voltage across the capacitor to an incremental change in time.

$$i_{Cav} = C(\Delta V_C/\Delta t)$$

E. If the rate of change is linear (that is time changes uniformly), then current will be average change. Thus average current is the change in voltage over the change in time.

F. The *greater* the rate of change of voltage, the *greater* the current.

V. Wave Shaping

A. Square wave input to the RC circuit.

B. Square wave circuit and waveform *shown in class*

C. If the RC time constant is short, the output of V_C is almost a square wave.

D. If the RC time constant (τ) is long the waveform is complex, and is similar to the charge/discharge curve.

E. Work Examples 10.5, 10.6 with a square wave as the source

VI. Capacitors in Series

A. Capacitors, like resistors may be placed in series.

B. The charge in coulombs is the same on each capacitor.

$$Q_T = Q_1 = Q_2 = Q_3 \text{ etc.}$$

C. The current in a series circuit is the same through out the circuit.

D. Kirchhoff's Voltage Law still applies to the voltage drop across the series capacitors.

$$E = V_1 + V_2 + V_3$$

E. Recall that $V = Q/C$

$\therefore\ Q_T/C_T = Q_1/C_1 + Q_2/C_2 + Q_3/C_3$

Since the charge (Q) is the same for all the caps, the divide equation by "Q".

$\therefore\ 1/C_T = 1/C_1 + 1/C_2 + 1/C_3$

For two caps in series:

$C_T = C_1C_2/(C_1 + C_2)$

F. When capacitors are connected in series, the ratio between any two potential differences is the inverse ratio of their capacitances.

$Q = C_1V_1,\ \ Q = C_2V_2$

$C_1V_1 = C_2V_2 \quad \Rightarrow \quad C_1/C_2 = V_2/V_1$

G. The basic effect of capacitors in series is similar to increasing the distance between the positive plate of the first capacitor and the negative plate of the last capacitor.

H. Capacitors in series use the parallel resistance equation form.

VII. Capacitors in Parallel

A. The total charge supplied by the source is equal to the sum of the individual charges of capacitors in parallel.

$Q_T = Q_1 + Q_2 + Q_3$

B. The voltage drop across each capacitor in parallel is the same.

$E = V_1 = V_2 = V_3$

Again $\quad Q = CV$

$\therefore C_TE = C_1V_1 + C_2V_2 + C_3V_3$ Divide equation by the voltage (E or V)

Results

$\therefore C_T = C_1 + C_2 + C_3$

C. Capacitors in parallel result in an increase in the plate area.

D. Capacitors in parallel use the series resistance equation form.

E. Total capacitance of caps in parallel is equal to the sum of the individual capacitances.

E. Work examples 10.14, 10.15, Problem 10-45.

VIII. Energy stored by a capacitor

A. Ideally a capacitor stores energy in an electric field.

B. As a capacitor is charging, energy is expended by the source of emf.

1. Some energy is converted to heat in the circuit resistance

2. It is a changing storage, as the current decreases in the capacitor, the voltage across the capacitor increases.

$p = vi$ also $W = Pt$ (W is energy)

$W = (Vav)(I)(t)$ and $It = Q$, $Q = CV$, and $Vav = V/2$

$W = V/2(CV)$ $W = CV^2/2$

C. Work is energy stored in the capacitor and expressed in *joules*. The area under the curve is the energy stored by the capacitor.

IX. Applications and characteristics of capacitors in DC circuits

A. Voltage doubler/tripler etc., used to step a voltage up to a much higher value. However, the current goes down proportionally. (See Flash Camera in Text.)

B. The storage of energy by a capacitor provides filtering in a DC power supply which is made from a rectifier circuit.

C. Large current spikes for a short duration of time will not damage the capacitor or the source of emf. The cap absorbs this energy, a surge protector.

D. The energy stored in a capacitor can leak or bleed off in some capacitors.

E. Capacitors are used as a timing device along with resistors.

F. Capacitance can come from sources other than capacitors. Stray capacitance occurs in parallel wires in circuits, between the junctions of semiconductors such as transistors. Usually these are not a factor at low frequencies or DC, but can be a problem at very high frequencies.