

Class Notes: Tesla Coils and the Failure of Lumped-Element Circuit Theory

by

Kenneth L. Corum and James F. Corum, Ph.D.

© 1999 by K.L. Corum and J.F. Corum

"The working [of the Extra Coil] was excellent with 1/4 wave-length." Nikola Tesla, September 18, 1899

I. Introduction. Can one model the physical operation of a Tesla coil appropriately with only lumped-element circuits? If not, why not? It was pointed out long ago that, *at its operating frequency*, a Tesla coil is **not** a lumped-element induction coil. Forget the quest for "many turns of fine wire". In fact, a Tesla coil has more in common with a cavity resonator than it does with a conventional inductor. [See TCTUTOR, Corum, 1988, pp. 56-58; "Extra Coil as a Slow Wave Resonator," Proc. 1986 ITS Symposium, pp. 2:1-2:24; Industrial Electron Accelerators, by E.A. Abramyan, Hemisphere, 1988, p. 94.] With a real Tesla coil, voltage rise is **neither** by lumped-element transformer action ($E_2 = NE_1$), **nor** by induction ($E_2 = M di/dt$), **nor** by simple lumped-element coupled resonance [$V_2 = V_1 \times (L_2/L_1)^{1/2}$]. In all of those circuit models the *current* is analytically presupposed to be *uniformly* distributed along the wire in the coil (it's in the Neumann integral definition of inductance - see any elementary electromagnetics text) and the voltage will rise proportional to N, the turns along the coil. There are no standing waves on a lumped element circuit component. (In fact, lumped-element circuit theory inherently employs the cosmological presupposition that the speed of light is infinite, as every EE sophomore should know. See, e.g., - Electric Circuits, by J.W. Nilsson, Addison-Wesley, 1983, p. 3.)

However, a true Tesla coil (circa 1894) is a velocity inhibited slow-wave helical *transmission line resonator*: $V_{\max} = S \times V_{\min}$, where S is the standing wave ratio. *Voltage magnification is by standing waves*. Period. No such voltages, even in the remotest degree, can be obtained by either lumped element transformers or by lumped element LC resonating circuits. This behavior of all quarter-wave resonators is well known. [See "Resonant Lines and Radio Circuits," by F.E. Terman, Trans. AIEE, July, 1934, pp. 1046-1053; Hyper and Ultra High Frequency Engineering, by R. Sarbacher and E.W. Edson, Wiley, 1943, p. 353; Networks, Lines and Fields, by J.D. Ryder, Prentice-Hall, 1949, p. 285; Electromagnetic Waves & Radiating Systems, by E.C. Jordan and K. Balmain, Prentice-Hall, 1968, pp. 227, 231.] In the following note, we will show why one needs transmission line analysis (or **Maxwell's equations**) to model these electrically distributed structures. Lumped circuit theory fails because it's a *theory* whose presuppositions are inadequate. Every EE in the world was warned of this in their first sophomore circuits course. (A pre-discharge, linear operating regime is being assumed for both lumped and distributed operation, of course.)

It makes no difference whether the coil is a cylindrical helix, a conical frustum, or a flat spiral. Tesla clearly understood the velocity-inhibited nature of spiral and helical resonators and taught that, "The length of the wire coil in each transformer should be approximately one-quarter of the wave length of the electric disturbance in the circuit, this estimate being *based on the velocity of propagation of the disturbance through the coil itself.*" [US Patent 645,576; Applied for Sept. 2, 1897] It's not the physical length of the wire but rather the velocity inhibited *electrical length* of the helical coil which must be quarter-wave resonant (i.e., have forward and reflected wave-interference producing a standing quarter-wave resonance). This was recognized by Tesla, and this is the meaning of the phrase "...this estimate being *based on the velocity of propagation of the disturbance through the coil itself.*" (The consideration could not exist for a lumped element, obviously.) Berkeley physicist David Sloan, ("An RF High Voltage Generator," Phys. Rev., Vol. 47, 1935, pp. 62-70), did not know how to mathematically handle the inhibited velocity of propagation on the helical resonator, and a suitable engineering analysis was provided only a decade ago.

Is there any question that Tesla is speaking of a distributed transmission-line resonator? Well, if so, listen to his correspondence to the US Patent Examiner on November 15, 1897. He is explaining what happens if the resonator excitation-frequency is raised (i.e., the wavelength shortened). As every electrical engineer knows, a grounded quarter-wave transmission-line resonator possesses a V_{\min} at the base and a V_{\max} at the top. If the frequency is lowered, the structure is too short for quarter-wave resonance, and if the frequency is raised, then V_{\max} positions form down in the resonator. The same is true for both helical coils and spiral coils. Concerning the latter, Tesla wrote,

"If the transmitting and receiving coil were made longer than the quarter of the wave-length of the electrical disturbance in the wire, then the points of highest potential would not fall at the inner *ends* of the coils ... as required, but *nodal points* would form, as the case may be, somewhere in the *middle* of the coils ..."

[Dr. Nikola Tesla - Selected Patent Wrappers, compiled by J.T. Ratzlaff, Tesla Book Company, 1980, Vol. 1, p. 150.]

This phenomenon is decisive. It occurs only on *distributed* resonators: it is impossible with any lumped circuit element! (The *current* has the same value at every point along a lumped-element.) To understand what is happening, consider a cylindrical helical coil of height H . The base is always forced to be a voltage node (it's grounded). The top is always a relative voltage loop at the odd quarter-wave resonances and a voltage node at the even (half-wave) resonances. These boundary conditions constrain the mode patterns on the structure (called spatial harmonics). We assert that velocity inhibited *partially coherent* forward and reflected RF traveling waves form interference patterns on the coil.

II. Physical Description. At the fundamental (quarter-wave) resonant frequency there is a V_{\min} at

the base and the V_{\max} appears at H (the top). At the next mode there's a V_{\min} at the base, a V_{\max} appears at $1/3 H$, then there's another V_{\min} at $2/3 H$, and, finally, a V_{\max} at the top. (The structure is 3 quarter-wavelengths tall.) At the next resonant mode there are voltage nodes at the base, $2/5 H$, and $4/5 H$; and a V_{\max} appears at $1/5 H$, $3/5 H$, and at the top. (The structure is 5 quarter-wavelengths tall.) To see the nodes sharply, you have to be at the resonant frequencies. (The impedance is following a circle around the Smith chart and the voltage is varying accordingly.) We will number the modes in terms of the number of quarter-waves on the structure: $n = 4H/\lambda_g$. [The resonant frequencies are called "overtones" instead of "harmonics". Only overtones that are integer multiples of the fundamental are called harmonics. For a *nondispersive* resonator the fundamental is called the first harmonic, the first overtone is the second harmonic, the second overtone is the third harmonic, etc. However, *a helix is a frequency dispersive resonator* - the velocity factor is *not* a linear function of frequency and, therefore, the overtones of the resonator are *not* integer multiples of the fundamental.] At the next overtone, the voltage nodes are at the base, $2/7 H$, $4/7 H$, and $6/7 H$. The V_{\max} are at $1/7 H$, $3/7 H$, $5/7 H$ and the top. Sketch a vertical line with 7 equidistant tick marks. Sketch heavy dots (nodes) at 0, 2, 4, and 6. Sketch in loops with maxima at 1, 3, 5 and the top. (The structure is 7 quarter-wavelengths tall.) The magnitude distribution of the spatial interference pattern is called a standing wave. At all the odd resonant overtones, there is always a V_{\max} at the top and a V_{\min} at the base. The pipe organ, trombone, violin, harp, guitar, xylophone, ... (even a flag pole) ... are all transmission line resonators. No wonder Helmholtz and Lord Kelvin were so entranced by Tesla's wonderful coil: it's a musical instrument whose very soul has been tuned for creating an electrical fountain of celestial fire. One can write analytical expressions for all this, of course. (They're in Appendix VI of our book Vacuum Tube Tesla Coils.)

III. An Experimental Test. In spite of the fact that Tesla, himself, concurs, all the above are just assertions until an experiment is conducted. Well, here is a **simple test** to see if *your* Tesla Coil is operating as a lumped circuit or a distributed circuit, i.e. - to see if it's possible to use lumped circuit theory to analytically model your coil. Stand your resonator up (in monopole fashion), connect a signal generator between its base and ground (you could connect the base directly to ground and link couple the generator to the coil), and sweep the oscillator up through the resonant frequencies. Hold an oscilloscope probe near the top and make a note of the frequencies (resonances) that show a V_{\max} . If other loop (V_{\max}) and node (V_{\min}) positions form down the coil at the higher resonant frequencies - congratulations. You're beholding a transmission line resonator. You'll see the voltage loop and node positions move down as the frequency is raised. (You could, if so disposed, measure the current along the coil. There will be a current *maximum* at the base and a current *minimum* at the top of a distributed resonator: ***the current entering one end of a tuned transmission line resonator is not equal to that exiting at the other end*** - this isn't DC!)] If either the current diminishes at the top of the coil or the loops and nodes migrate down the coil, then you have a *distributed element*, and *lumped circuit modeling fails*. Period. Lumped circuit theory isn't absolute truth, it's only an analytical *theory* - and in these resonators we have the case where this sophomore *theory* fails *experimentally*. The engineer must either use Maxwell's equations or distributed elements to model reality. (If lumped analysis describes your coil, cheer up - modify its operation to an open resonator and you'll see what Tesla called, on July

11, 1899, "a beautiful advance in the art"! Helical resonators of this genre are common knowledge in the engineering community. Top loading by a sphere or toroid will foreshorten the resonator in a predictable way. (TCTUTOR, p. 50) The procedure given by Tesla is to select a top loading (toroid or sphere) that is *physically* large enough to prevent discharge prior to reaching the desired potential, and then design the resonator to operate with this load reactance at a frequency that brings the top-loaded system into resonance at the desired potential. (CSN, July 11, 1899) This is different than designing resonant lumped element systems, where currents are uniform along coils and voltage rises are *much* smaller. It was documented (analytically *and* experimentally) over a decade ago, and it's astonishing that it's still disputed seriously by some Tesla aficionados.

As with all experiments, be careful to control stray effects such as the mutual capacitance between the scope probe leads and the resonator. (You're looking for spatial variations in a geometrically compact field distribution.) Even the experimenter's body will modify the resonator's voltage distribution. We constructed a sliding coax-fed E-field probe. (It's a variation of the conventional *slotted-line* experiment so familiar to undergraduate EEs.) This simple probe samples the resonator's external E-field (proportional to the voltage distribution along the coil) without seriously perturbing the fields, and it avoids errors that would arise from the experimenter waving his arms near the coil. (An H-field probe could be used, if desired.) A simple procedure (we'll call it **Test I**) to observe transmission line modes on a Tesla Coil is:

1. Tune to the fundamental frequency and observe the V_{\max} at the top and the V_{\min} at the base.
2. With an oscilloscope probe (or even a neon bulb) near the top, tune to the first *overtone* (the next frequency where the V_{\max} at the top again reappears sharply). Then move the probe downward along the coil. You will see a V_{\min} (near the 2/3 H point) and a V_{\max} (near the 1/3 H point). And then, of course, a V_{\min} again at the base. [*Lumped element coils can't do this.*]
3. To convince yourself that the transmission line resonator theory is really gospel, tune to the 5th V_{\max} observed at the top (counting the fundamental as 1). The structure is then 9 quarter-wavelengths tall. There will be 5 V_{\max} s, with a guide quarter-wavelength equal to the coil length divided by 9. You can either move the probe up and down the coil to find the 5 separate voltage maxima, or you can place the probe 1/9 up and swing through the V_{\max} . (Place the probe 2/9 up and you'll sweep through a voltage null at the same frequency.)

[There is even enchantment for the analytically minded. If you solve Maxwell's equations on a helix for the interior and exterior fields, and match the boundary conditions radially across the helix, you will be led to a transcendental equation which must be solved iteratively for the guide phase constant from which the all important inhibited velocity factor may be determined. (It will be on the order of 1/1000 or less for a Tesla helix.) The spatial "wavelength compression factor"

for the helical transmission line resonator then follows, as do the *predicted resonant frequencies* and the V_{\max} positions. This all sounds complicated, but it's not, really. And, no - there is no violation of the equation of continuity (i.e., conservation of charge) on helical resonators - even though the *current* is **different** at the two ends (the base and the top).]

We ran the experiment on the coil shown in Photo 1, below. The geometrical parameters of the coil were as follows: $N = 317$ turns, $D = 24.3$ ", $H = 55.4$ ", #10 gauge stranded copper (1 kV insulation), $d_w = 0.1019$ ", $s = 0.175$ ". The predictions and measured data were:

Mode	Predicted Vmax Position	Measured Vmax Position	Predicted f_o (kHz)	Measured f (kHz)	Error ($\Delta f / f_o$)
1	55.4"	Top (55.4")	180	175	2.8%
3	18.5"	18.0"	455	435	4.4%
5	11.1"	11.0"	677	645	4.7%
7	7.9"	8.0"	903	860	4.8%

The normalized measured voltage wave interference patterns for the first three resonant frequencies were plotted alongside the coil as shown in Photo 1. The experiment was also conducted on a small helical coil with the following parameters: $N = 532$ turns, $D = 6.3$ ", $H = 25$ ", #18 gauge enameled, $d_w = .0403$, $s = .047$ ". The data are tabulated below and provide added confirmation of the transmission line resonator theory.

Mode	Predicted Vmax Position	Measured Vmax Position	Predicted f_o (kHz)	Measured f (kHz)	Error ($\Delta f / f_o$)
1	25"	Top (25")	540	520	3.7%
3	8.33"	8.0"	1,210	1,240	2.5%

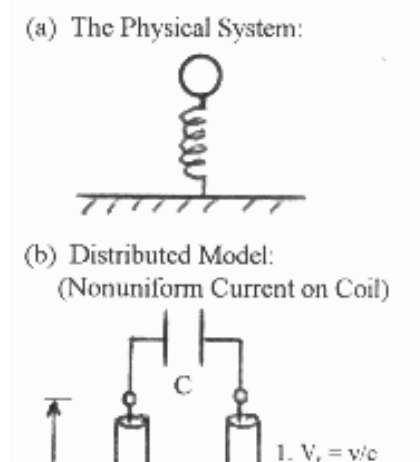
5	5.0 "	5.0 "	1,753	1,800
2.7%				
7	3.6 "	3.5 "	2,250	2,350
4.4%				
9	2.78 "	2.75 "	2,750	2,800
1.8%				

The theory and experiment agree to within 5%, which is acceptable engineering accuracy. Anybody should be able to obtain similar results.

IV. What's Going On? All those handbook formulas that people use for inductance, L , inherently *assume* applications at frequencies so low that the current distribution along the coil is uniform. (They were all derived from the Neumann integral with an assumed *uniform* current, which, historically, is an attempt to *geometrically* characterize a coil in terms of a ratio of magnetic flux linkage to impressed current.) The real issue is that migrating voltage nodes and loops *are not a property of lumped-circuit elements* - they are the directly observable consequence of velocity inhibited wave *interference* on the self-resonant coil. Lumped element representations for coils **require** that the current is uniformly distributed along the coil - no wave interference and no standing waves can be present on lumped elements. The problem has been that many experimenters working with self-resonant helices have *pursued the concept of coil self-capacitance without really understanding where the notion comes from or why it was ever invoked by engineers*. For that, they will have to go read R.W.P. King's wonderful old book *Electromagnetic Engineering*, McGraw-Hill, 1945. (See pp. 418-422, 461-466.) On pg. 465, the Harvard Professor points out that, for coils whose **wire length** exceeds $1/6$ wavelength (as is the case for the secondary of authentic Tesla coils), **"an adequate representation of the reactance of a coil with a nonuniformly distributed current is not possible in terms of a coil with a uniform current [a lumped-element inductance] connected in parallel with a lumped capacitance."** Period. Resonant *fields* present surprises to engineers with limited training.

V. Discussion. Figure 1 shows an actual physical system and two candidate models: (1) a transmission line (distributed element) model (that allows one to include the effect of a spatially varying current distribution); and (2) a lumped-element model. Depending upon whether you're operating at a frequency where the current is sinusoidally distributed along the coil or operating at a low enough frequency so that the current is uniformly distributed along the coil, either model may be appropriate. (*Distributed theory encompasses lumped circuits and always applies.*)

In the **transmission line resonator model**, the ball on the top serves as a load reactance conjugate to the transmission line back impedance to bring the system, of electrical length $\theta =$



$\beta_g l$, into resonance at the operating frequency and, assuming no discharges, the voltage rise is proportional to the VSWR on the resonator. Smith charts are the easiest way to work with distributed circuits. For a resonant system, enter the chart at the capacitive load and proceed along a VSWR circle to the V_{\min} at the input end. When the losses go to zero in the distributed resonator mode, S goes to ∞ and the voltage magnification is limited only by the avalanche breakdown around the top electrode. The goal is to make the structure's losses small and, therefore, keep the VSWR as high as possible - just the opposite of what RF engineers usually want to do with VSWR! The voltage magnification can be truly stunning for distributed resonance. ("*There is sweet music here.*" - Tennyson)

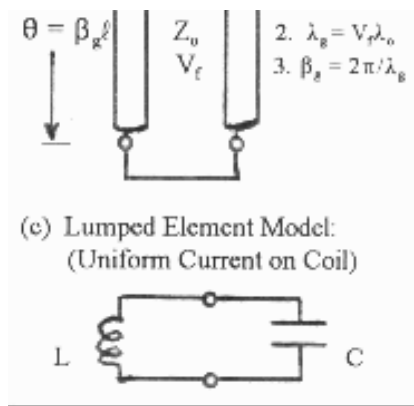


Figure 1. Physical System and Models.

In the **lumped element model**, the sphere serves as a conjugate reactance, equal and opposite to $2\pi f_0 L$, where L is the self inductance of the coil measured at (or calculated for) *frequencies so low that the current distribution on the coil is uniform*. A thorough analytical investigation of such *tuned* coupled lumped resonance was given in TCTUTOR (p.16-44). In the lossless case, the voltage across the secondary is given by the conservation of energy expression $V_2 = V_1(L_2/L_1)^{1/2} = V_1(C_1/C_2)^{1/2}$, RF magic which Tesla had discovered at his Grand St. Laboratory in 1891 and disclosed to Hertz at Bonn in 1892. (See Vacuum Tube Tesla Coils, Appendix X.) This is the most lumped elements can give - even if the coil has no resistance whatsoever! When real world losses in the coil are included the voltage step-up is even smaller than this optimistic $(C_1/C_2)^{1/2}$ expression. The voltage magnification in coupled tuned coils is nowhere near what is possible from a simple distributed resonator with standing waves.

VI. Operating Regimes. How can you tell whether your Tesla coil is operating in Tesla's pre-1894 tuned lumped element regime or in his post-1894 distributed resonator regime? You could just look at the field distributions, as described in the test above. Here's a decisive set of quantitative measurements to see *how bad* the lumped-element model really is. Do the experiment on your Tesla coil. We'll call it **Test II**.

Step 1. With the sphere (or toroid) attached on the top of your coil *and a ground connected to its base*, link couple a signal generator into the bottom of the coil and hold up an oscilloscope probe in the vicinity of the sphere - but not so close as to interfere with the experiment. Sweep the signal generator until the first V_{\max} is observed. *Call this the system resonant frequency, f_o .* **Step 2.** With the coil disconnected, but the top capacitor supported at the same height above ground as when it's on the coil, measure C. **Step 3.** Calculate the capacitive reactance of the top capacitor at the measured system resonant frequency f_o : $X_C(f_o) = 1/(2\pi f_o C)$. **Step 4.** With L and C still disconnected, measure the self inductance of the coil (without the top capacitor) with an LCR bridge at 1 kHz (i.e., at a frequency so low that the current on the coil will be uniformly distributed). Call this value L. **Step 5.** Calculate the frequency at which the lumped-element inductance has the same reactance as the actual capacitive reactance in steps 1 and 3. That is, $f = X_C(f_o)/(2\pi L)$. If $f \sim f_o$ then the coil in step 1 was operating in the lumped-element regime.

Well . . . What frequency did you get in step 5? Is it the same as f_o ? Is the difference within engineering accuracy, i.e. is $|f-f_o|/f_o \times 100\%$ less than 5%? If the answer is yes, then you may confidently use lumped-element modeling. *However, if the answer is no*, then, from the halls of Valhalla, old Wotan, *himself*, is thundering out over the battlements, "#*@&%!! . . . Thor, you dum dum! You *can't* use lumped circuit modeling! . . ." [The coil has standing waves and is behaving as a distributed resonator.] Now, which model describes the operation of *your* coil? We took the coil shown in Photo 1 and top-loaded it with a 25 pF (22" by 8") toroid and performed **Test II**. The results were:

Step 1. $f_o = 135$ kHz (The toroid load lowered the resonant frequency and foreshortened the helical resonator to $\theta = 65.7^\circ$.) **Step 2.** $C = 25.6$ pF (measured at an elevation of 70") **Step 3.** $X_C(f_o) = 46,052$ ohms **Step 4.** $L = 0.022$ H (measured with a BK Precision LCR meter, model 875A) **Step 5.** $f = 333.155$ kHz (So, $|f - f_o|/f_o \times 100\% = \underline{\underline{146.8\%}}$ error. . . *Not even close* to being a lumped-element circuit!)

By the way, TCTUTOR predicts that, for $C = 25.6$ pF, the operating frequency will be at $f = 140$ kHz, and the error is only 3.7%.

About a decade ago, while one of the authors was serving as a Senior Research Scientist at the Battelle Memorial Institute's Columbus, Ohio Laboratories, he had the opportunity to acquire Bill Wysock's **Tesla Technology Research Model-10** commercial Tesla Coil. (See TCBA, 1983 Vol.

2, #3, p. 21, and the [TTR website](#)). We have the coil's parameters and measured data before us: $N = 342$ turns, $D = 2.0'$, $H = 97.8''$, $d_W = 0.102''$ (silver stranded #10), $s = 0.286''$. The Model-10 is capped with a large 4 foot diameter spun aluminum toroid. (See Photo 2.) The results of **Test II**, to determine the appropriateness of "lumped-element" coil modeling, are as follows:

Step 1. $f_o = 140$ kHz (measured) **Step 2.** $C = 48$ pF (measured) The 4 foot diameter toroid is supported 13 feet above ground. **Step 3.** $X_C(f_o) = 23,684$ ohms **Step 4.** $L = 16.3$ mH (measured with an HP-4262A bridge at 1kHz) **Step 5.** $f = 231.25$ kHz. The "lumped-element assumption" has $|(f - f_o)|/f_o \times 100\% = \underline{65.2\%}$ error. . . . (Lumped circuits? . . . *Not even close.*)

Transmission line modeling of this coil predicts that, for $C = 48$ pF, the operating frequency will be $f = 133$ kHz. The "distributed-element" model's error is only 5%. (Maxwell's equations win again!)

Photo 2 is a 1990 snapshot when we first turned the coil on. We were limited by the fact that the lab (the old foundry), though several hundred feet in length, was only 50' wide and 25' high. (The discharges hit the walls and ceiling.) The Model-10 has a professionally constructed system control panel housed in a 7' mobile relay rack cabinet, with all kinds of instrumentation, heavy duty switching components, safety interlocks, remote control, etc. The safety protected 35 kV power transformer was modestly rated at 25 kVA and the coil (with appropriate loading) was advertised as capable of 6 megavolt peaks. We routinely used it (at half throttle) for 30 foot directed discharges. (At night the flash and echo were awesome!) When the author left Battelle, this marvelous machine was rescued by the Ohio State University's Electrical and Computer Engineering Department, and, since 1995, it has been prominently displayed in the Department's modern High Voltage Lab facility adjacent to Drees Lab, where it continues to be used in high voltage

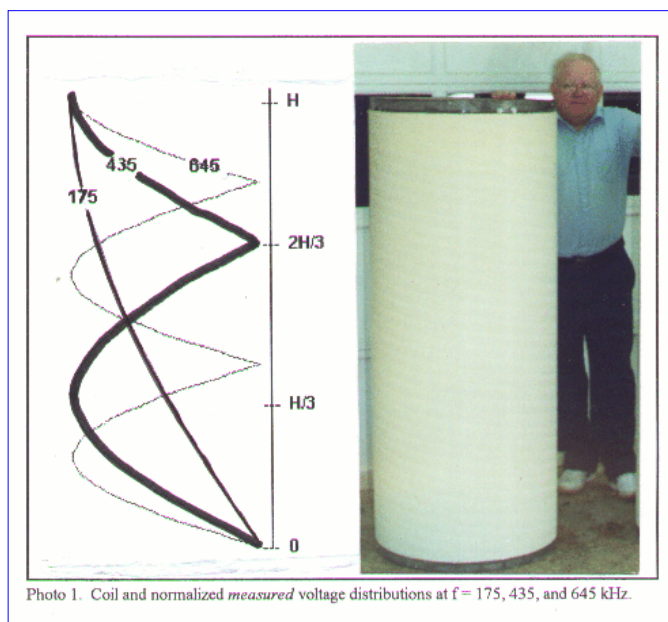
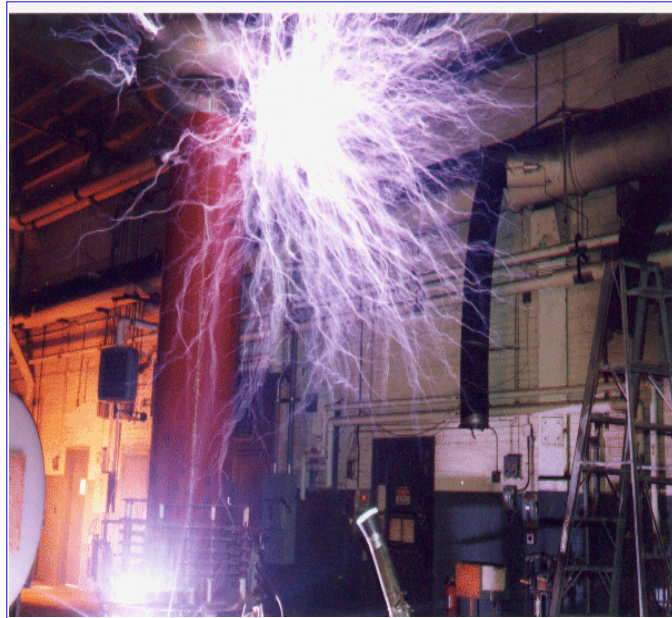


Photo 1. Coil and normalized measured voltage distributions at $f = 175, 435,$ and 645 kHz.

research at OSU. (Go Bucks!)

Photo 1. Coil and normalized *measured* voltage distributions at $f=175, 435,$ and 645 kHz.



The OSU High Voltage Lab web page can be found at

<http://eewww.eng.ohio-state.edu/~sebo/hvlab.html>.

In May, 1999, the authors were pleased to visit at Ohio State with Professor Stephen Sebo, the Lab's Director, and operate the coil once again. We digress. The conclusion to be drawn from Test II is that modeling with lumped circuits is of marginal utility (it's a naive swindle!) for really comprehending any modern Tesla coil or understanding Tesla's post 1894 research.

Photo 2. Bill Wysock's *TTR Model-10* as initially set up at Battelle.

Several years ago, a numerically modeled transient analysis of coupled lumped-element coils was published in the AJP ("A Solid-State Low Voltage Tesla Coil Demonstrator," by D. Bruns, American Journal of Physics, Vol. 60, 1992, pp. 797-803). An *exact* mathematical analysis of the same circuitry, *including losses*, had been given in TCTUTOR back in 1988 when we pointed out that a lumped analysis is appropriate only during the duration of the primary spark (while the coupled flux is uniform throughout the resonator), and we indicated the great importance of controlling the primary switching epoch. (Finkelstein's optimum $k = 0.6$ criteria is valid only if the spark duration is roughly $1/(2\Delta f)$ where Δf is the line splitting passband broadening of the over-coupled circuit.) **That the coupled-coil, lumped-element model of Tesla coils is physically fallacious** (it doesn't predict the three-humped spectrum that is actually observed when the spark duration is finite) **has been known since the 1911 experimental observations of Dr. Fleming.** ("Some Resonance Curves taken with Impact and Spark-Ball Discharges," by J.A. Fleming and G.B. Dyke, Proc. of the Physical Soc., London, Vol. 23, 1911, pp. 136-146 (see comments by Dr. Eccles and Prof. Howe, p. 144); Also see "100 Years of Cavity Resonator Development," by J.F. Corum and K.L. Corum, Proc. 1990 ITS Symposium, pp. 2:1-18, Figs. 3, 4.) Needless to say, the 1992 AJP article is little more than the simulated impulse response of an IF can - the kind of tuned lumped coupled circuit Tesla was using before his splendid discovery of 1894. On the other hand, *if you* have been able to model *your* coil with a lumped-element inductance, L , with less than 5% error, then you've been working with a lumped, tuned, coupled-circuit (like a radio receiver IF can) just as Tesla was doing *prior* to 1894, before he discovered the true Tesla coil. [If you've got

a huge capacitance on the top then the length of wire in the secondary may be less than $\lambda_0/(2\pi)$ and, as voiced by Professor King, the current distribution could be *uniform enough* so that you can represent the transmission line as a lumped element. But, as Tesla once told his attorney, "**A large capacity and a small self inductance is the poorest kind of circuit which can be constructed.**" The good news is: there's a whole new realm waiting for you to explore. "*Oh brave new world that has such creatures in it!*" (Tempest, V.1)

VII. Closing Thoughts. There is nothing sacred about using a sine wave generator (a finite power signal) to perform the above tests. The more sophisticated experimenter might want to repeat **Test II** with a broadband white noise (stochastic) source coupled to the resonator with an untuned link. Fundamental and overtone resonances can be observed with a calibrated receiver or a spectrum analyzer. Those with a knowledge of modern optics will easily recognize even richer phenomena occurring. (It can be shown that the ultimate limit in voltage rise on the coil is set by the degree of coherence of the up and back resonator waves.) While this may shatter the egos of some coilers, contrary to popular myth, the *key performance parameter* for a high voltage Tesla coil is *not the length of the discharge* (which is a function of things like input energy and primary spark duration) but, rather, the **VSWR** on the resonator coil - *the higher the better!* [It's related to the fringe "Visibility Function" for quasimonochromatic, partially coherent optical beams. (Principles of Optics, by Born and Wolf, 5th ed., p. 506.)]

The tests above are experiments that EEs have commonly performed since the '30's in undergraduate courses associated with RF electronics, and they are easily replicated with trifling effort. Concerning idealized circuit elements, Prof. Ron Scott has written that students shouldn't be "disappointed to learn that **circuit theory is not real engineering.**" (Linear Circuits, Addison-Wesley, 1960, p. 2.) Finally, we point out that virtually *all modern Tesla Coils* are velocity inhibited, distributed-element, slow wave transmission line resonators. Those asserting the contrary simply have not done their homework. Tesla said that he discovered this striking nature of coils in 1894, "That was *the first single step* toward ... my magnifying transmitter." [Tesla on His Work with AC, edited by L. Anderson, Sun Publishing, 1992, p. 72.]

Perform steps 1-5 of **Test II** on your coil. And then do **Test I**, mapping the voltage distributions at the fundamental and overtones. Now go back and reread Tesla's 1897 patent application comment, above. His remarks also apply in the case when "the impressed oscillations are more rapid than the free oscillations" (Patent # 1,119,732), but that's about electric fire-balls and is enchantment for another time.

Jim Hardesty has made many of the Corum papers on Tesla coils available through the PV Scientific Instruments web page at www.arcsandsparks.com.